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Radius of influence for a cosmic-ray soil moisture probe: Theory and Monte Carlo simulations

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Radius of influence for a cosmic-ray soil moisture probe: Theory and Monte Carlo simulations

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Abstract

The lateral footprint of a cosmic-ray soil moisture probe was determined using diffusion theory and neutron transport simulations. The footprint is radial and can be described by a single parameter, an e-folding length that is closely related to the slowing down length in air. In our work the slowing down length is defined as the crow-flight distance traveled by a neutron from nuclear emission as a fast neutron to detection at a lower energy threshold defined by the detector. Here the footprint is defined as the area encompassed by two e-fold distances, i.e. the area from which 86% of the recorded neutrons originate. The slowing down length is approximately 150 m at sea level for neutrons detected over a wide range of energies–from 10^0 to 10^5 eV. Both theory and simulations indicate that the slowing down length is inversely proportional to air density and linearly proportional to the height of the sensor above the ground for heights up to 100 m. Simulations suggest that the radius of influence for neutrons >1 eV is only slightly influenced by soil moisture content, and depends weakly on the energy sensitivity of the neutron detector. Good agreement between the theoretical slowing down length in air and the simulated slowing down length near the air/ground interface support the conclusion that the footprint is determined mainly by the neutron scattering properties of air.

CONTENTS

FIGURES

1. INTRODUCTION

One of the major challenges in land-surface hydrology is the determination of soil moisture content across a range of scales (Bloschl 2001). Soil moisture can vary considerably over short distances of just a few meters, making it difficult to accurately determine spatially averaged soil water content with point measurements unless large numbers of samples are used (Jacobs et al. 2004), or stationarity of the moisture field can be assumed. Although satellite retrievals of microwave brightness temperature can be used as a proxy for soil moisture averaged over tens of kilometers, even when taken at face value the application of these data at smaller scales requires complex and uncertain disaggregation procedures that rely on less accurate radar mode or other auxiliary data (e.g. Reichle et al. 2001; Merlin et al. 2008). An alternative method for field scale observations is the cosmic-ray technique (Zreda et al. 2008; Desilets et al. 2010), which (in static mode) offers ground-base observations at a scale intermediate between point and satellite measurements. In dynamic (roving) mode the technique can potentially provide regional and continental scale measurements with sub-kilometer resolution (e.g. Desilets et al. 2010).

Cosmic-ray neutron fluxes can be used to non-invasively determine soil water content or snow water equivalent depth to depths of tens of decimeters (Zreda et al. 2008; Desilets et al. 2010). The technique utilizes cosmic-rays as a natural source of neutrons in air and soil, and relies on the exceptional neutron scattering properties of hydrogen for its sensitivity to water. For example, a mineral soil containing only 0.05 $cm^3\ cm^{-3}$ of water will typically derive more than 50% of its moderating power from water because of the relatively large elastic scattering cross section and large energy loss per collision for hydrogen. Fast neutrons are emitted from nuclei through the evaporation process following excitation by secondary cosmic rays. Evaporation neutrons unleashed near the land surface are then moderated through collisions in the ground and atmosphere until they are thermalized or captured by nuclei. Neutron intensity in the fast to epithermal energy band is inversely proportional to water content.

The scale represented by non-invasive neutron measurements is largely controlled by the diffusive properties of air. For the cosmic-ray probe, scale refers to the average distance that neutrons travel between being moderated in soil and detected somewhere above the land surface. If neutrons are detected at a significantly lower energy than they are emitted from the ground, then neutrons must first suffer multiple collisions in air prior to detection. Because the mean free path for each collision is on the order of several tens of meters in air, the neutron signal would be expected to originate from within a radius extending to several hundreds of meters. The mean lifetime of a neutron in air is less than 100 ms, and the mean downscattering time is less than 1 ms (Carron 2007), making the scattering and diffusion of neutrons in air nearly instantaneous.

The aim of this paper is to provide a theoretical framework and quantitative analysis of the factors governing the radius of influence. The approach here is to first build an analytic framework describing the mean travel distance through the atmosphere of neutrons emitted from soil. The derived relationships are used to parameterize the results of neutron transport simulations, which involve fewer implicit simplifications regarding the complexity of the scattering process. Despite several potentially limiting simplifications, the analytically derived radius of influence is actually in good agreement with the results of the more physically rigorous but less insightful neutron transport simulations. Theory and simulations are used together to constrain important characteristics of the cosmic-ray probe footprint, including how it depends on soil moisture, neutron energy, barometric pressure, height above the ground.

2. THEORY

2.1. Radius of influence defined

Here we use elementary diffusion and slowing down theory to derive an expression for the radius of influence for a neutron detector located on a planar source of neutrons. The radius of influence defines a laterally extensive area that contributes an arbitrarily large fraction of the counting rate observed by a detector. We use diffusion theory because the scattering of neutrons by nuclei is analogous to stochastic interactions between gas molecules. We use slowing down theory because source cosmic-ray neutrons are not initially at thermal equilibrium with their surroundings, but must be moderated to lower energies through elastic collisions with soil or air nuclei.

In the foregoing analysis we show with diffusion theory that the radius of influence for neutrons emanating from a plane source can be described by a single parameter, an e-folding length. One e-folding length is equivalent to the radius encompassing the source area for 100x[1-e^{-1}]% of the counts. We begin with the assumption that a planar neutron source can be treated as being made up from a number of point sources, with the resulting neutron flux given by the superposition of the point sources (Glasstone and Edlund 1952).We first consider the flux distribution surrounding a point source, then obtain the flux at the center of a ring source, and finally integrate the rings to obtain the flux at a point from a planar source.

According to diffusion theory, the neutron flux ϕ [n L^{-2} T^{-1}] around a point source in an infinite, homogenous medium, varies with the distance *r* [L] according to (Glasstone and Edlund 1952).

$$\phi(r) = \frac{S_{\text{pt}}}{4\pi D}\frac{e^{-\frac{r}{L}}}{r^{m}} \qquad 1$$

where S_{pt} is the source intensity [n T^{-1}], *D* is the diffusion constant [L^{-1}] and *L* is an e-folding length [L] describing neutron absorption. The factor $\exp(-r/L)$ accounts for neutron absorption resulting from collisions with air nuclei, and the factor $1/r^n$ accounts for spreading of the flux from the point source. For neutrons behaving diffusively, where the loss of outgoing neutrons is partially compensated by a backscattered flux, *m*=1 (Glasstone and Edlund 1952).

The intensity of neutrons emitted from an infinitely thin annulus, S_{ring} [n T^{-1}], is given by

$$S_{\text{ring}} = S_{\text{pt}} 2\pi r \qquad 2$$

and therefore the neutron flux at the center of the ring, ϕ_0, is

$$\phi_0(r) = \frac{S_{\text{ring}}}{4\pi D}\frac{\mathrm{e}^{-\frac{r}{L}}}{r} = \frac{S_{\text{pt}}}{2D}\mathrm{e}^{-\frac{r}{L}} \qquad 3$$

which can be rewritten as

$$\phi_0(r) = C\mathrm{e}^{-\frac{r}{L}} \qquad 4$$

where $C = \frac{S_{\text{pt}}}{2D}$. 5

To obtain the intensity at any point on the surface of an infinite plane source, Equation 4 can be integrated over a continuum of rings extending to infinity

$$\int_{\infty}^{0} \phi_0(r)dr = C\int_{\infty}^{0} \mathrm{e}^{-\frac{r}{L}}dr = \frac{C}{k} \qquad 6$$

and the intensity of neutrons originating within distance $r = L$ is given by

$$\int_{L}^{0} \phi_0 = C\int_{L}^{0} \mathrm{e}^{-\frac{r}{L}}dr = \frac{C}{k}\left(1 - e^{-1}\right) \qquad 7$$

The probability that a neutron originates from within $r = L$ is then given by

$$\frac{\int_{L}^{0} \phi_0(r)dr}{\int_{\infty}^{0} \phi_0(r)dr} = 1 - e^{-1} \qquad 8$$

This equation shows that the e-folding length L is equivalent to the radius circumscribing an area contributing 100x[1-e^{-1}]% of the total contribution to a point detector located on an infinite plane source. We define the radius of influence to be the distance corresponding to two e-fold increases in the contribution to the count rate, i.e., the radius encompassing the source area for 86% of the recorded neutrons, which the above analysis proves is equivalent to two e-folding lengths.

2.2. Radius of influence as a slowing down length

In elementary diffusion theory, the value of L is calculated from the absorption properties of the medium. This concept can be adapted to the present case, in which neutrons are primarily eliminated by slowing down rather than absorption. In other words, a neutron in a given energy bin is effectively absorbed when it is moderated to an energy below the lower threshold of the bin. To determine the value of the corresponding e-folding length, we take advantage of an alternative but mathematically equivalent definition of the e-folding length from diffusion theory which gives *L* as the e-folding length as the mean distance (mean free path) traveled by a neutron before being absorbed (Glasstone and Edlund 1952). In the case of slowing down, *L* can be taken as the mean distance between emission and moderation to below a threshold value. This distance can be determined from particle tracking simulations, as in Section 3, or by calculating the mean radial distance traveled by a neutron from its point of emission from the ground to point of detection within some space above the ground, as in the following section.

2.3. Slowing down length calculated for an infinite, homogenous atmosphere

The radius of influence for land-surface neutron measurement can be described by the mean distance traveled between emission and moderation to detection energy. We refer to this distance as the slowing down length L_s and point out that our definition is more general than the definition conventional to neutron diffusion theory, which usually refers to the slowing down length as the distance between emission and thermalization (Glasstone and Edlund 1952). In this paper the slowing down length is the mean "crow-flight" distance traveled by a neutron while being moderated from an initial energy of emission, *E*, to a final energy of detection, *E'*. For analytical convenience we assume that *E'* corresponds to the median sensitivity of the detector, although in practice neutrons are usually detected over a range of energies.

The slowing down length can be calculated from root-mean-square displacement $\langle r^2 \rangle$ of a neutron while slowing from *E* to *E'*. According to diffusion theory

$$\langle r^2 \rangle = \left(\lambda \sqrt{n}\right)^2 \qquad 9$$

where is the jump length is λ and *n* is the number of collisions. The number of collisions required on average to slow a neutron from *E* to *E'* is given by

$$n = \frac{\eta}{\xi} \qquad 10$$

where η is the number of log decrements between *E* to *E'*

$$\eta = \ln E - \ln E' \qquad 11$$

and ξ is the log decrement energy loss per collision, given by (Glasstone and Edlund, 1952)

$$\xi = 1 + \frac{(A-1)^2}{2A} \ln \frac{A-1}{A+1} \qquad 12$$

The effective atomic weight for a mixture of elements can be calculated from

$$A_{\text{eff}} = \frac{\sum_{i=1}^{n} A_i \lambda_i^{-1}}{\sum_{i=1}^{n} \lambda_i^{-1}} \qquad 13$$

The value for air is A_{eff} = 14.25 g mol^{-1}, which yields η = 7.5 collisions on average to reduce the neutron energy by one natural logarithmic decrement. The jump length is equivalent to the collision mean free path, calculated from

$$\lambda = \frac{1}{\Sigma_s} \qquad 14$$

Where Σ_s [cm^{-1}] is the macroscopic scattering cross section, calculated from

$$\Sigma_s = \sigma_s \frac{A}{\rho N} \qquad 15$$

where A [g mol^{-1}] is atomic mass, ρ [g cm^{-3}] is density, N [atoms mol^{-1}] is Avogadro's number and σ_j [cm^2] is the elastic scattering cross section for reaction j. For a mixture of elements with i=n components:

$$\lambda = \sum_{i=1}^{n} \sigma_{s,i} \frac{A_i}{\rho_i N_i} \qquad 16$$

The slowing down length is then calculated from the mean square displacement (Equation 9) according to

$$L_s^2 = \frac{1}{6} \left\langle r^2 \right\rangle \qquad 17$$

There are two additional complexities that must be incorporated into the simple diffusion treatment outlined above. One is that the elastic scattering cross section increases with decreasing energy. This means that the jump length increases with increasing energy, as shown in Figure 1. To account for this, we calculate the average values for the jump length over logarithmically spaced bins. If the effective jump length for energy bin i is given by the logarithmically-weighted average value λ_i, where i=1 corresponds to the first log decrement of energy loss after neutron emission, then the total root-mean-square displacement over all log decrements can be estimated from:

$$\left\langle r^2 \right\rangle = \sum_{\mathrm{i}} \lambda_i \left(\sqrt{i} - \sqrt{i-1} \right) \sqrt{n_i} \qquad 18$$

Because the average number of collisions experienced by a neutron before detection increases with initial energy, the slowing down length increases with increasing initial energy.

A second complexity is that neutrons are emitted from soil over a range of energies. This is because neutrons are generated with initial energies governed by the nuclear evaporation process, which is statistical in nature, and because neutrons generated at depth lose energy through random elastic collisions in soil before escaping. If the detection energy is centered on 10 eV, approximately the median value for a slow neutron detector moderated with 1-3 cm of plastic, and the initial energy is assumed to range from the evaporation peak at 2×10^6 eV down to the detection energy, then the slowing down length will range from approximately 240 m for neutrons emitted at 2×10^6 eV to 0 m for neutrons already at the detection energy. The mean slowing down length for all emitted neutrons should be between these two values, with the precise value depending on the shape of the emission spectrum.

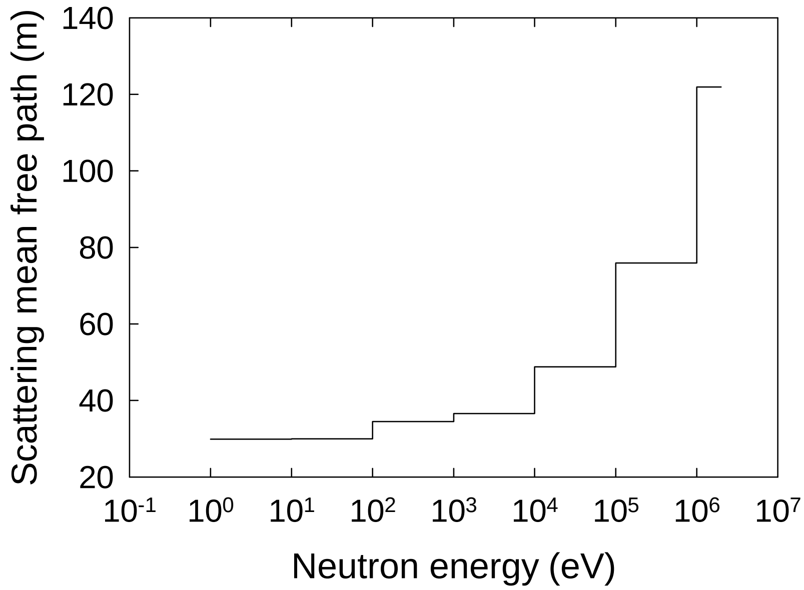

Figure 1. Neutron scattering mean free path as a function of energy at sea level for neutrons traveling in air at sea. Calculated using cross sections for nitrogen and oxygen from ENDLVII.

We used the transport code Monte Carlo N-Particle 5 (MCNP5) to do determine the emission spectrum of land surface neutrons. Our model utilizes a source function that decreases with an e-folding length of 150 g cm^{-2} and an evaporation source spectrum peaked at 2×10^6 eV. Fluxes are tallied as a function of energy in an above-ground slab as neutrons leave the soil. The atmosphere is modeled as a vacuum to prevent neutrons from diffusing back into the detector layer after entering the atmosphere. Our results, shown in Figure 2, indicate that above 10^4 eV the differential spectrum is not in equilibrium with slowing down and therefore varies significantly from the 1/*E* equilibrium predicted by slowing down theory. The importance of this process in governing the fast neutron spectrum has been verified by simulation and measurement of the secondary cosmic-ray neutron flux (Goldhagen et al. 2004). The implication is that the proportion of fast neutrons to slow neutrons is significantly greater than would be predicted by an equilibrium slowing down spectrum.

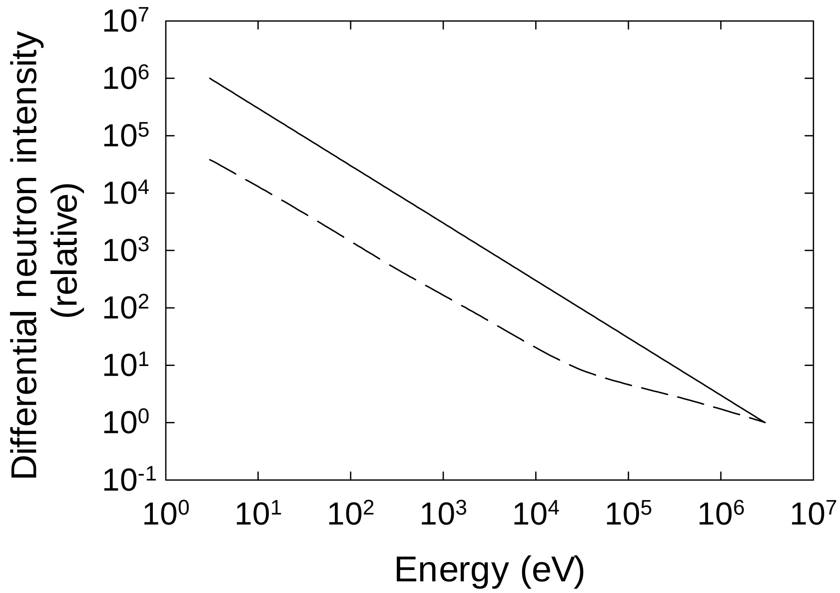

Figure 2. Emission spectrum simulated in MCNPX (dashed) for soil compared to theoretical 1/*E* slowing down spectrum (solid).

Using our simulated spectrum, the average slowing down length is given by

$$\langle L_s \rangle = \frac{\sum_i L_{s,i} w_i}{\sum_i w_i} \qquad 19$$

where $L_{s,i}$ is the slowing down length for a neutron with initial energy E_i and the weights w_i are calculated from

$$w_i(E) = \phi_{E_i} \left(\ln(E_i) - \ln(E_{i-1})\right) \qquad 20$$

where ϕ_{E_i} is the logarithmically averaged differential neutron flux at energy E_i. Using the corrected spectrum, the slowing down length averaged from $E_i = 2\text{x}10^6$ eV to $E_i = 10^2$ eV is 149 m. This length is approximately the average crow-flight distance traveled in the atmosphere by a neutron from emission to the point of detection at 10^2 eV.

Although the theory outlined above is a gross approximation, it does yield several useful insights into the factors governing the radius of influence. First, it suggests that the footprint is directly proportional to the collision mean free path, which is inversely proportional to atmospheric density. The radius of influence should therefore increase in direct proportion to air density. Second, the theory implies that the footprint should increase with the separation between the energies of emission and detection due to the larger number of collisions involved. Third, the radius of influence should grow in proportion to the square root of the number of collisions. This implies that early collisions are responsible for greater net displacement than later ones. A corollary is that the net displacement does not increase much with additional collisions after the first several collisions. And finally, if our assumption that transport is dominated by the properties of the atmosphere is correct, then differences in soil moisture content should not affect the radius of influence. In the next section these findings are evaluated more rigorously using a full Monte Carlo simulation of neutron transport near the land surface.

3. MONTE CARLO SIMULATIONS

We use the code Monte Carlo N-Particle Version 5 (MCNP5) to overcome limitations inherent to diffusion theory (Pelowitz 2005). With the MCNP5 code individual particle histories are simulated by sampling probability distributions that describe the source energy, source direction and results of all subsequent interactions including scattering and absorption. Advantages of the Monte Carlo approach include the ease of incorporating complex boundaries, better accuracy near boundaries, continuous-energy treatment of interactions, and flexible tallying options. Because diffusion theory is inherently unreliable near the boundaries of different materials, a major advantage for the work described here is the ability to track particle interactions near the land/air interface.

For computational efficiency, we start with the assumption that a spatially distributed source can be treated as a collection of point sources. We can therefore track particles from a point source and integrate over a circle to create a finite plane source. This strategy allows us to track particles from a point source over a wide area, which is vastly more efficient than using an aerially expansive source and tracking only the particles that reach a point detector or small volume detector. As shown in Section 2, the mean distance traveled by a neutron from a point source while being moderated to E' is equivalent to the radial distance from a point detector for 1-1/e of the neutrons emitted from a plane source at E'. Although diffusion theory shows that these two problems can both be described by the same e-folding length, it does not provide a rigorous way to calculate that length. For this we apply Monte Carlo simulations.

The goal of our simulations is to track the distance traveled by a neutron from the point of emission from the ground to the point of detection in the atmosphere. We assume that a neutron will be emitted from soil at an initial energy E but must be moderated to energy E' to be detected. As in our diffusion model, each random collision will tend to carry the neutron farther from the point of emission. However, in our MCNP we specify in that in order to be detected the neutron must also be within two meters of the land surface, which is where detectors are typically installed (e.g. Zreda et al. 2008; Desilets et al. 2010). Neutrons that are within two meters of the soil but have not crossed the threshold *E'* are not tallied.

The energy of emission from the surface is not directly specified in our model. It is instead determined by sampling an evaporation source spectrum and by transport through soil. This is accomplished by employing a line source that extends along the z-axis from the land surface to the bottom boundary of the modeling domain. To simulate the attenuation of neutron-producing cosmic ray flux as it penetrates the subsurface, the source intensity in our model decreases exponentially with depth according to an e-folding length of 160 g cm^{-2}. The initial neutron energy along this line is randomly sampled from an evaporation energy spectrum (Pelowitz 2005) with a central energy of $2x10^6$ eV, which corresponds approximately to the evaporation peak measured with a multisphere neutron spectrometer (Goldhagen et al. 2004). Because neutrons are first emitted from the ground within a lateral radius of only 0.5 m on average from where they originated on the line source, on the scale of hundreds of meters this can be considered approximately as a point source on the land surface.

The modeling domain is a box with sides 2000 m apart and a height of 7645 m. The subsurface is 4 meters deep and is modeled as a pure SiO_2 with a bulk density of 1.4 g cm^{-3}. The atmosphere is 7641 meters thick and contains 78% N and 22% O with a density that ranges from $1.2x10^{-3}$ g cm^{-3} at the ground surface to $0.6x10^{-3}$ g cm^{-3} at the top. We typically run at least 200,000 particles per simulation, of which typically 2-3% meet the criteria for detection. Most of the remaining 97-98% of the particles are either transported to heights greater than 2 m above the ground or to deep in the soil by the time they reach the detection threshold *E´*.

Simulations were performed at varying moisture contents, air densities and for different heights of a detector above the ground. For a detector above the ground surface, fluxes were tallied in a 2 m layer centered at a selected elevation above the ground. This layer is referred to as the detector layer. We tracked the coordinates of all particles within the detection energy range that entered the detector layer. These are referred to as detected neutrons, and the coordinates of entry give the point of detection.

4. RESULTS AND DISCUSSION

We used the particle tracking feature in MCNP to record the coordinates of all nuclear collisions experienced by particles meeting detection criteria. In the way of examples, Figure 3 shows particle tracks for three random walks generated with our model. Tracks A and B demonstrate that numerous collisions can occur in air before a neutron is detected near the ground. Trajectories A and C demonstrate that a track may intersect the ground a second time after a neutron is first emitted.

The trajectories in Figure 3 illustrate that the first few collisions in air account for most of the total displacement from the source. This behavior is implied by diffusion theory (Equation 9), which indicates that the mean displacement grows in proportion to the square root of the number of collisions. The first collisions also tend to give greater displacement because the mean free path between collisions increases with energy.

We calculated the radius of influence from our Monte Carlo simulations in three different ways. One way is by calculating the mean of radial displacement for all "detected" particles. The radial displacement is the distance traveled from the source before making a final entry into the detector layer at energy *E´*. This gives the range of the neutrons but does not account for secondary interactions that might occur between emission and detection. Our simulations show that a neutron intersects the ground on average 2.0 times before being detected. Such interactions would also contribute to the signal and could distort the real sample area. We therefore also calculated the radius of influence a second way, using the mean distance of each collision in the detector layer for neutrons of any energy. The collision density should be proportional to the neutron flux. Because collisions in the detector layer should be a good proxy for collisions in the nearby ground, this approach is intrinsically sensitive to secondary interactions near the land surface after emission, but unlike the first method does not necessarily indicate the final coordinates for detection since in or model detected neutrons need not collide with air in the detector layer. Finally we calculated the radius of influence from the mean distance of the final collision in the detector layer for all "detected" neutrons.

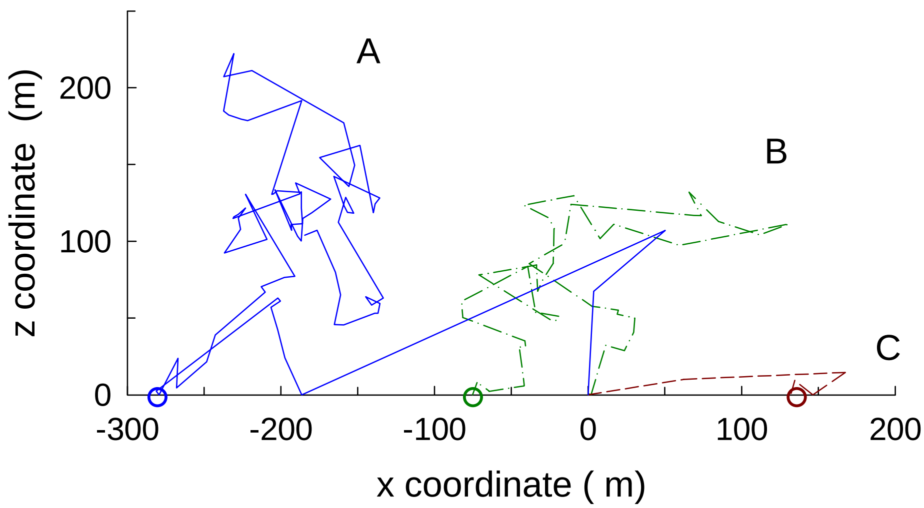

Figure 3. Three random walks for particles emitted from the ground surface and detected within 2 meters of the ground.

The three methods yield a nearly identical (to within 1%) effective slowing down length of 150 m for dry land at sea level. Although secondary interactions with the ground are common, the agreement shown here suggests that secondary collisions with soil do not affect the radius of influence. How is this possible? This result implies that the distance between each secondary soil interactions and the point of detection (or final interaction) is on average equal to the distance from the point of detection to the point of emission.

The effective value of 150 m also agrees well with the theoretical value derived in Section 2. Assuming an E' is 10^2 eV, the theoretical slowing down length (using the modeled emission spectrum) is 147 m in air. The good agreement between theory and simulations suggest that the radius of influence is determined mainly by the properties of air, and only weakly on soil water content. Because the interaction mean free path increases with energy, and because the crow flight distance is proportional to the square root of the number of collisions (Equation 9), the first few collisions in air account for most of the displacement.

4.1. Dependence on moisture content

Our simulations indicate that the radius is only about 7-5% smaller over saturated soil compared to dry land. The direction of the change is consistent with the suggestion that secondary interactions with wetter ground rapidly moderate neutrons thereby limiting their range, making the footprint smaller. The small size of the change is consistent with a radius of influence that is determined mainly by the transport properties of the atmosphere, which do not change nearly as much over time as the transport properties of soil.

4.2. Dependence on energy

We have calculated the radius of influence over dry soil as a function of energy (Figure 4). In agreement with theory, the footprint tends to increase with the separation between the detection and emission energies (decrease in detection energy), although not as strongly as theory would suggest. The radius of influence actually decreases with energy at the lowest energies (10^{-2} to 10^{-1} eV range). We suggest as an explanation that the high thermal neutron absorption cross strong for nitrogen plays a role. Strong absorption of thermal neutrons by atmospheric nitrogen tends to favor the survival of thermal neutrons moderated in and emitted directly from the ground over those moderated in the atmosphere. Since there are on average fewer collisions (less moderation) in the atmosphere, the radius of influence is smaller.

Although the radius of influence tends to increase with decreasing detection energy, the change is small over a broad plateau between 10^0 to 10^5 eV. This indicates that differences in detector energy sensitivity in this range will not significantly change the footprint. Detectors with different energy sensitivities can therefore be used to monitor the same sample area. For example, simultaneous observations of fast and slow neutrons can be used to determine the presence of snow without the complication of mismatched sample areas.

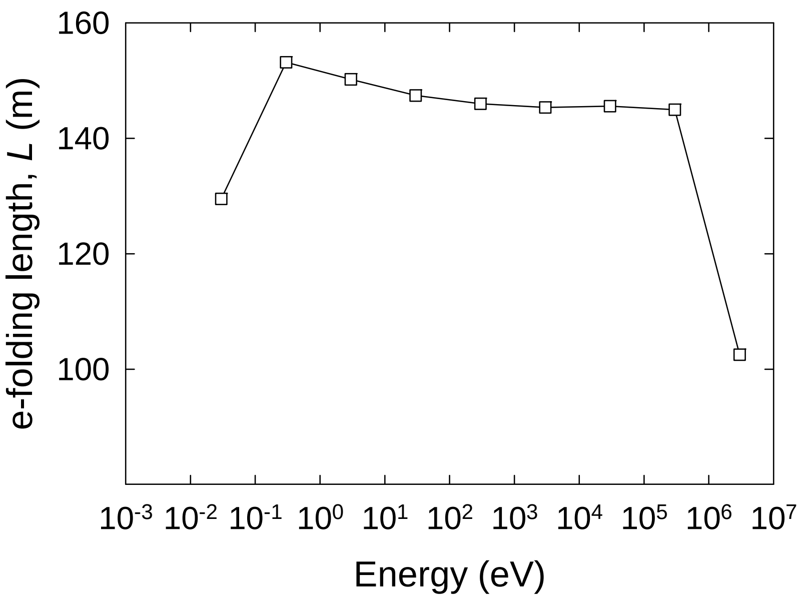

Figure 4. E-folding length as a function of energy at detection for ground based detector.

4.3. Dependence on atmospheric pressure

Because the scattering mean free path in air depends on atmospheric density, which is an inverse function of elevation, the slowing down length should increase with elevation. If the slowing down length is governed almost entirely by the properties of air, as our simulations suggest, then the radius of influence should be inversely proportional to atmospheric pressure. The slowing down length, L_s at pressure P is given as:

$$L_s = L_{s,0}\left(\frac{P}{P_{s,0}}\right) \quad 21$$

where $L_{s,0}$ and $P_{s,0}$ are reference values. To test the validity of this equation, we calculated the slowing down length for different atmospheric densities using MCNP5 (Figure 5). Our simulations indicate that Equation 21 is accurate, and support the observation that the properties of air dominate the simulated slowing down length.

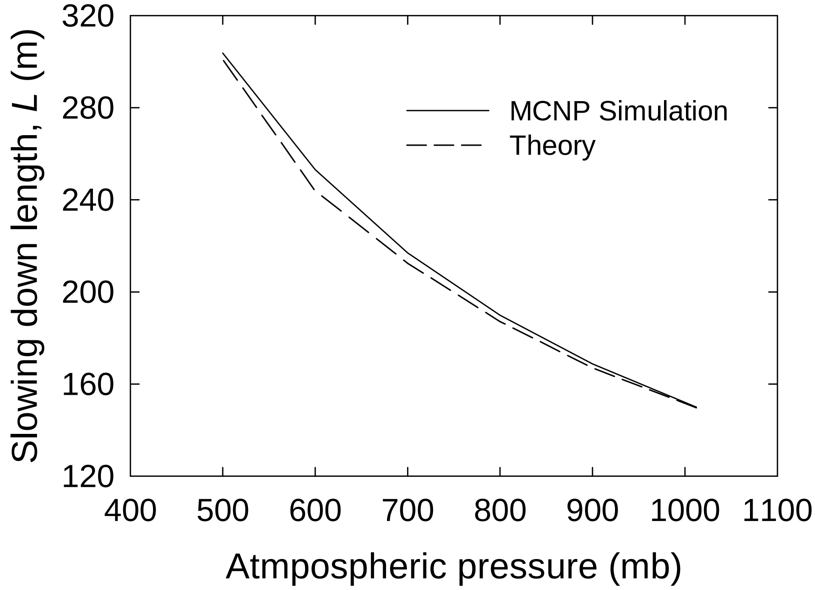

Figure 5. Slowing down length as a function of atmospheric pressure.

4.4. Dependence on height above ground

Diffusion theory indicates that the radius of influence should increase with height above the ground. A simple geometric relationship can be used to estimate this effect. Neutrons emitted from a point source travel an average distance L_s in the atmosphere while being moderated to the detection threshold E'. Collisions in air for 1-1/e^{-1} of the detected neutrons can be envisioned as taking place within the perimeter a half sphere extending into the atmosphere and defined by radius equal to L_s (Figure 6). If the neutron flux distribution is governed only be the slowing down length L_s, then it follows from purely geometrical relationships that the radius of influence for a detector should increases by an amount equal to the distance from the ground. The radius of influence is therefore larger than $2L_s$ for a detector above the ground. However, the neutron flux distribution is still described by an exponential function. We call the corresponding e-folding length L_s^*, which is given by

$$L_s^*(h) = L_s + h \qquad 22$$

where L_s is the slowing down length at ground level and h is height above the ground.

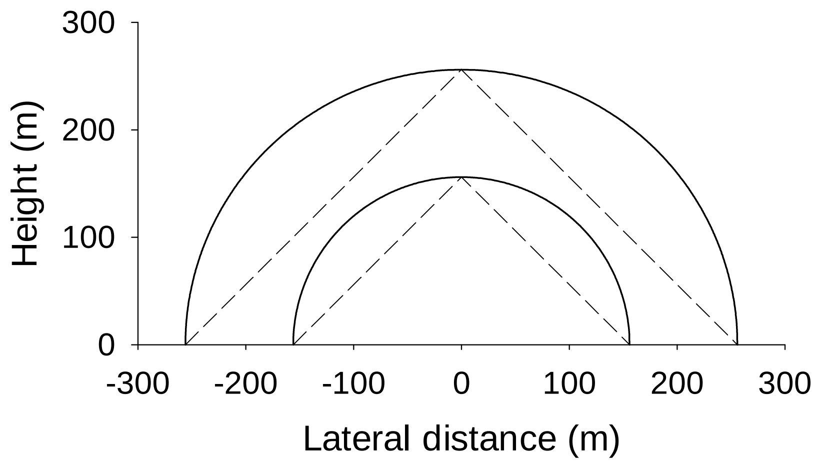

Figure 6. Diagram of half sphere showing the influence of height on the radius of influence.

We simulated the height effect in MCNP by calculating the slowing down length from particle tracking. The slowing down length is calculated from the mean travel distance from the point of emission to the point of detection for a 2 m thick detector layer located at an average height above the ground ranging from 1 to 200 m depending on the simulation. Coordinates were tracked for all collisions within the detector layer. Our results, shown in Figure 7, indicate that the radius of influence is described accurately by Equation 22 up to a height of 100 m. Although the radius of influence should in principle continue to increase above 200 m the technique is probably not usable at higher detector heights due to the attenuation in air of neutrons emanating from the ground.

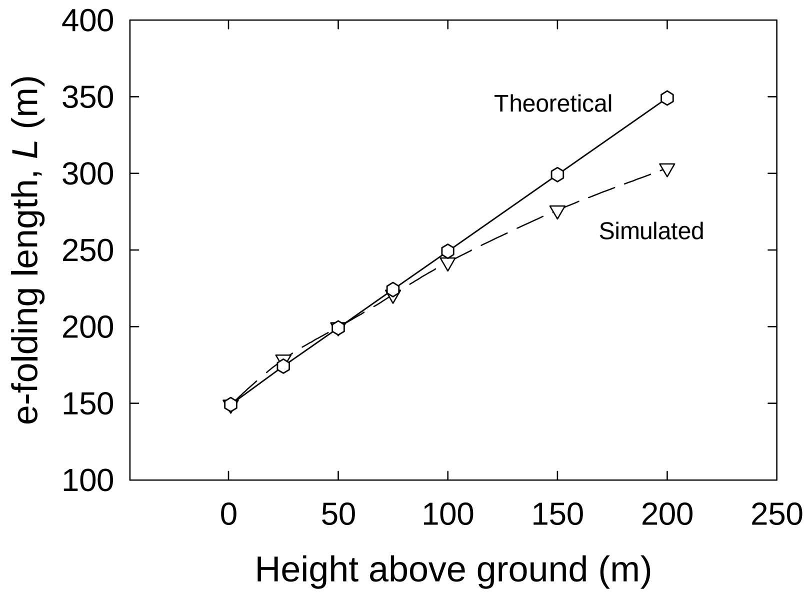

Figure 7. Slowing down length as a function of height from theory and MCNP simulations.

5. SUMMARY AND CONCLUSIONS

We used diffusion theory and Monte Carlo simulations to quantify and parameterize the radius of influence for a cosmic-ray probe. Our work indicates that:

(1) The radius of influence can be described by a single parameter: the exponential e-folding length *L*;

(2) The e-folding length describing the slowing down of neutrons in air (L_s) is about 150 m at sea level for neutrons detected at energies of 10^2 eV. This gives a radius of influence of 300 m for the area contributing 86% of the counts;

(3) The radius of influence depends mainly on the properties of air, and therefore is insensitive to the soil moisture content or soil chemistry;

(4) The radius of influence for E >1 eV increases in proportion to the square root of the number of collisions required to moderate a neutron to detection energy, and therefore with the separation between emission energy E and detection threshold E';

(5) The radius of influence increases is inversely proportion to air density and therefore increases with elevation;

(6) For heights of up to at least 100 m, the radius of influence increases with height above the ground for according to:

$$L_s^*(h) = L_s + h$$

where $L_s^*(h)$ is the slowing down length at height *h*.

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