FINAL TECHNICAL REPORT

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TITLE: Approaches to Some of the Outstanding Problems of Heterogeneous Compactive Deformation of Geomaterials

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PROJECT DESCRIPTION

Evidence from laboratory experiments and field observations on porous rocks (and other porous materials) has indicated that compaction does not necessarily occur homogeneously, but, instead, is localized in narrow planar zones that are perpendicular to the maximum compressive stress. Because the permeability of these zones is reduced by several orders of magnitude, they present barriers to fluid flow across them. Consequently, their formation in reservoirs or aquifers can adversely affect attempts to inject or withdraw fluids, such as $CO_2$. Because the zones are narrow, they will be difficult to detect from the surface and, as a result, it is important to understand the conditions for their formation and extension.

## Propagation of Compaction Bands

Observations of the lengths of compaction bands in the laboratory are limited to the width of the specimen, but bands in the field are of the order of tens of meters in length. Part of our work has focused on understanding the conditions governing the extension of the bands. The most detailed field study of the bands to date [Sternlof, 2006; Sternlof et al., 2005] indicates that the bands have a roughly elliptical profile with aspect ratio of $10^{-3}$ to $10^{-4}$. These observations suggest a model of a band as a narrow ellipsoidal inclusion subjected to a uniform inelastic compactive strain. For this model the compactive displacement of the boundary is proportional to the width of the band and supports the use of the width as proxy for the band displacement, as done in field studies. A detailed parameter study of this model indicates that the stress states within and near the edge of the band are well-approximated by the limiting case of zero aspect ratio and relatively insensitive to elastic mismatch of the band and surrounding material, so long as it is not too large [Rudnicki, 2007].

A log-log plot of Sternlof's data with a few other data points from the field (Figure 1) indicates that the midpoint width of the band scales as $w = AL^{\alpha}$ where $A = 0.08558\ \text{mm}^{1/2}$, $\alpha = 0.49891$ and $L$ is the half-length of the band [Rudnicki et al., 2006]. This roughly square root dependence on the length is consistent with the response of a very thin elliptical inclusion subjected to uniform compactive displacement $w$ over the central portion of the band $-a < x < a$. A uniform traction equal to the difference between the farfield compression and the resistance to closure due to the band material is specified over the remainder of the band, $|a| \le x < |L|$. The magnitude of this traction is taken to ensure that the compactive displacement varies smoothly along the entire band. If it is assumed that propagation of the band requires a critical value of energy released per unit area of advance, $G_{crit}$, then $w$ is proportional to $\sqrt{L}$ (for a fixed ratio of $a/L$). For small $a/L$, consistent with the field observations, the coefficient corresponding to $A$ is given to a good approximation by $\sqrt{8G_{crit}(1-\nu)/\pi\mu}$, where $\mu$ is the shear modulus and $\nu$ is Poisson's ratio. For values of $\nu$ ($0.2$) and $\mu$ ($8.3$ GPa), representative of Sternlof's field site, $G_{crit} = 30\ \text{kJ/m}^2$, comparable to other estimates from the field (10 to 60 kJ/m$^2$ [Rudnicki and Sternlof, 2005]) and from nominal stress strain curves of notched laboratory specimens (40 to 60 kJ/m$^2$ [Tembe et al., 2006]). The decrease of energy release rate $G$ (proportional to the square of the stress intensity factor at the tip of the band) with band length in this model suggests that the bands form with $G > G_{crit}$ and then grow until $G$ falls below $G_{crit}$.

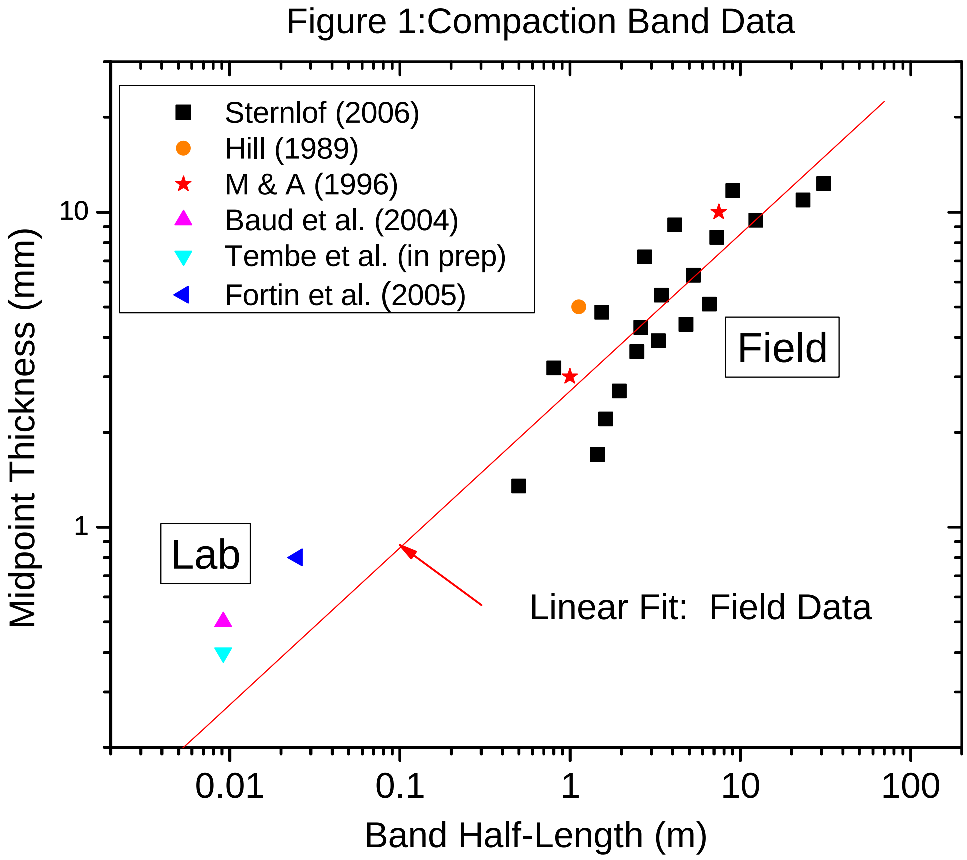
Figure 1:Compaction Band Data

Laboratory values of $w$ and $L$ data plot above the best-fit line for the field data in Figure 1. Consequently values calculated for $G_{crit}$ for the laboratory data using the best fit line to the field data (and the same values of $\mu$ and $\nu$) are much higher than values inferred from the nominal stress strain curves (roughly 70 to 112 kJ/m$^2$). The higher values inferred from the model based on the field data are consistent with the interpretation that the bands propagate across the entire laboratory specimen before the energy release rate falls to its critical value.

## Results from True Triaxial Tests

Although treating shear and compaction band formation as a bifurcation from homogeneous deformation has proven to be a useful framework, the results are strongly dependent on the material constitutive relations that are used. The vast majority of laboratory tests on rocks are for axisymmetric loading of cylindrical specimens. Although such tests are useful, they provide limited information about the response to the full range of stress states that are encountered in applications. Although K. Mogi did pioneering work on true triaxial testing (all three principal stresses are different) in the 1960's, there has been little systematic work on this subject since then. Recently, however, Haimson and coworkers have conducted a series of true triaxial tests on several rock types. Comparison of their observations of failure angle with the theoretical predictions places constraints on the form of the constitutive model. In particular, the results suggest that neither the Mohr-Coulomb nor Drucker-Prager idealzations are adequate, and that more elaborate models are needed to describe the observations. A version of a three invariant model, similar to the Lade-Duncan and Matsuoka-Nakai models, has been successful in describing the observations on Westerly granite.

The results of shear band localization theory have been used to analyze true triaxial compression tests on specimens prepared from two siltstone core sections, one above and one below the Chelungpu Fault, Taiwan. In particular the results of the theory are used with fault angles observed for axisymmetric compression and deviatoric pure shear to infer properties of the inelastic constitutive behavior. These properties are significantly different for the two cores. Using them to predict fault angle $\theta$ for other deviatoric stress states yields good agreement with the observations for core II and acceptable agreement for core I. The results are used to predict the angle variation for constant mean normal stress ($\theta$ decreases as the deviatoric stress state varies from axisymmetric extension to axisymmetric compression) and at fixed deviatoric stress state ($\theta$ decreases monotonically with increasing mean normal stress).

## Effects of Coupling with Fluid Flow

Almost all the analyses and laboratory tests on compaction bands have been done under drained (or dry) conditions. In the field, compaction bands are thought to form under saturated conditions.

Analysis of the onset of localization (shear banding, faulting) as a bifurcation from homogeneous deformation (Bésuelle and Rudnicki, 2004) has proven to be a useful framework when applied to materials modeled as rate-independent. For saturated materials, fluid diffusion in response to deformation introduces rate-dependence even when the drained (constant pore pressure) response is rate-independent. Because only the instantaneous material properties enter the bifurcation condition, the application of this criterion to rate-dependent materials, including those for which the rate-dependence is due to fluid diffusion, is not productive. If, however, the drained response is rate-independent, then the alternative limit of undrained deformation, corresponding to no fluid mass change in material elements, is also rate-independent. For simple shear of a dilatant, pressure-sensitive material Rice (1975) has shown that homogeneous undrained deformation is unstable in the sense that small spatial nonuniformities grow exponentially in time when the localization condition is met in terms of drained response. Hence, if the localization condition is met in terms of the drained response before being met in terms of the undrained response, the latter is irrelevant. However, the condition for localization during undrained response may be met before or after the condition is met for drained response depending on the nature of the pressure sensitivity of yield and inelastic volume change.

For a wide class of rate-independent elastic – plastic material models with inelastic volume change and pressure-sensitivity, the form of the constitutive relation for undrained response is identical to that for drained response (Rudnicki, 2000). For the special case of elastically incompressible solid and fluid constituents, a good approximation for soils, the undrained response is pressure insensitive and inelastically incompressible even if the drained response is not. As a consequence of this form invariance, the localization condition for undrained deformation can be determined directly from the result for drained deformation by making appropriate substitutions. We have compared the localization conditions for drained and undrained conditions for a range of parameters describing the elastic compressibility of solid and fluid constituents, inelastic volume change, and dependence of the yield stress on mean normal stress. For undrained conditions, both the value of the critical hardening modulus for localization and the band angle depend on the parameters describing the poroelastic response. A particularly interesting case occurs when the inelastic volume deformation is compactive and the pressure sensitivity is positive. Such conditions may occur near the transition of a cap-like yield surface to a frictional yield surface and, thus, are relevant to compaction band (or compactive shear band) formation. For such cases, the localization condition for undrained response is met before that for drained response.

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