LLNL-TR-676586

Wall-Friction Support of Vertical Loads in Submerged Sand and Gravel Columns

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August 26, 2015

This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344.

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Summary

Laboratory studies of the 'floor-loads' under submerged vertical columns of sand and/or gravel indicate that such loads can be approximated by a buoyancy-corrected Janssen-silo-theory-like relationship. Similar to conditions in storage silos filled with dry granular solids, most of the weight of the sand or gravel is supported by wall friction forces. Laboratory measurements of the loads on the floor at the base of the water-filled columns (up to 25-diameters tall) indicate that the extra floor-load from the addition of the granular solid never exceeded the load that would exist under an unsupported (wide) bed of submerged sand or gravel that has a total depth corresponding to only two column-diameters. The measured floor-loads reached an asymptotic maximum value when the depth of granular material in the columns was only three or four pipe-diameters, and never increased further as the columns were filled to the top (e.g. up to heights of 10 to 25 diameters). The floor-loads were stable and remained the same for days after filling. Aggressive tapping (e.g. hitting the containing pipe on the outside, manually with a wrench up and down the height and around the circumference) could increase (and occasionally decrease) the floor load substantially, but there was no sudden collapse or slumping to a state without significant wall friction effects. Considerable effort was required, repeatedly tapping over almost the entire column wall periphery, in order to produce floor-loads that corresponded to the total buoyancy-corrected weight of granular material added to the columns. Projecting the observed laboratory behavior to field conditions would imply that a stable floor-load condition, with only a slightly higher total floor pressure than the pre-existing hydrostatic-head, would exist after a water-filled bore-hole is filled with sand or gravel. Significant seismic vibration (either a large nearby event or many micro-seismic events over an extended period) would likely be necessary before the full (buoyancy-corrected) weight of the sand and/or gravel would be 'delivered' to the bottom of the submerged column.

Background

This paper describes the findings from a laboratory study to address a practical engineering problem of predicting the solids-produced load at the bottom of a water-filled borehole when it is backfilled with gravel. During a recent series of tests involving buried explosives, a concern arose related to the load on an experiment canister containing explosives at the bottom of a vertical drill hole, and nearly 75m below the water table.

The field-scale system of interest consisted of a ~3-ft diameter (~0.9m) hole which was over 300-ft (>90m) deep, with the base over 250-ft (>75m) below the water table. The experiment canister at the bottom of the drill-hole was to be covered with sand, and then the hole was to be sealed with several meters of grout and filled with gravel before the explosive was ignited. In a similar previous test, however, heat from curing grout had contributed to some problems with the planned experiment. It was also estimated that if no grout was used the stress level that could be produced at the bottom of the hole from the weight of gravel added (if wall friction did not hold up the weight of the gravel) would exceed the experiment-canister design limit. The question of interest, then, was whether wall-friction, as occurs in silos filled with dry granular solids, would effectively support most of the weight of gravel used to back-fill the bore-hole. If wall friction would support most of the gravel's weight, then the grout could be eliminated (or relocated to any convenient height well away from the experiment canister) and the bore-hole could safely be filled with gravel to any height above the sand, without exceeding the design-level stress on the experiment canister at the bottom. Examination of this practical engineering problem has led to some interesting observations regarding stresses in granular columns which are reported below.

Review of Janssen's Differential Slice Analysis Approach for Floor-Loads in Silos

Janssen's analysis of floor-loads in silos [Janssen, 1895] has formed the basis for understanding how wall friction significantly alters the stress vs depth distribution in granular columns compared to the usual linear increase of pressure with depth (e.g., $P \approx \rho g h$, where ρ is density, g is the acceleration of gravity, and h is depth) that exists in fluids, or describes the usual average vertical stress in geologic beds. Variations from Janssen's predicted stress distribution are usually caused by non-symmetric configurations, dynamic conditions during silo emptying [Roberts, 2012], and/or variations in the state of compaction of the material within the silo (which can significantly modify the material's stress-strain response to small deformations). This study examines stresses in granular columns under conditions differing from the usual analyses of stresses in silos. Because the granular system of interest is under water, some differences from traditional silo-theory stress distributions are anticipated. For example, buoyancy reduces the effective density in the granular material; thus reducing all granular matrix stresses. Also, surface friction is often different for wet surfaces than for dry surfaces; and under submerged conditions, lubrication forces may affect the dynamic settlement of particles into the resulting packed bed in such a way that the fabric-matrix and/or the stress distribution in the particle-matrix might differ from that assumed in traditional silo analyses.

The following discussion is a brief outline of the assumptions and analysis that lead to Janssen's somewhat non-intuitive floor pressure relationship. Later, the effects of alternative boundary conditions and buoyancy-corrections are also included in a similar relation for floor loads in multi-layer, submerged vertical columns.

Referring to Figure 1 and following the logic of Janssen we consider gravity acting down with wall friction acting up, resisting the downward movement of the material. The granular material is assumed to be in a vertical cylindrical container of radius, R, with a wall friction coefficient, μ (for friction between the granular material and the pipe wall). Following Janssen, a few simplifying assumptions are made:

- The vertical stress is constant over a planar horizontal cross-section.
- The ratio between the horizontal and the vertical stress within the slice is a constant, $k = \sigma_h/\sigma_y$.
- The wall friction is 'fully developed' so that the vertical stress at the wall $\sigma_f = \mu\sigma_h$, and acts in a direction to resist relative motion between the pipe wall and the granular material.
- The bulk density, ρ, of the granular material is essentially constant throughout the bed.

A vertical force balance analysis on the differential slice produces a first order, ordinary differential equation for the vertical stress in the granular material (y is assumed to be zero at the top surface and increases with depth).

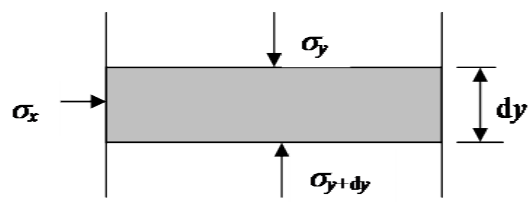


Figure 1– Differential slice for force balance analysis.

Force up = Force down

$$\pi R^2 \sigma_{y+\mathrm{d}y} + 2\pi R \mu \sigma_h \mathrm{d}y = \rho g \pi R^2 \mathrm{d}y + \sigma_y \pi R^2$$

rearranging and simplifying:

$$\frac{\sigma_{y+dy} - \sigma_y}{dy} + \frac{2\mu\sigma_h}{R} - \rho g = 0$$

or, noting that $\sigma_h = k\sigma_y$, we obtain,

$$\frac{d\sigma_y}{dy} + \frac{2\mu k}{R}\sigma_y - \rho g = 0 \; . \qquad (1)$$

Janssen's solution to this ODE is obtained if we assume that the stress, σ_y, is zero at the top free surface (where $y = 0$). The resulting vertical stress distribution solving Eqn (1) is,

$$\sigma_y = \frac{\rho g R}{2\mu k}\left(1 - e^{-\frac{2\mu k}{R}y}\right) \qquad (2)$$

where g = the acceleration of gravity. This is the well-known Janssen formula for floor-pressure due to material stored in a silo. Note that as y increases the exponential term vanishes, so that the maximum value of the vertical stress in the material (i.e., the floor-pressure of the material at the bottom of the vertical column), $\sigma_{y\,\max} = \frac{\rho g R}{2\mu k}$, scales with the radius of the silo, in contrast to pressure in a liquid which would scale linearly with the depth, and be independent of container radius.

Equation (2) has been verified for static granular beds in silos many times during the past 120 years. Figure 2 shows the Janssen theory, Eqn (2), and the measured vertical load at the base obtained in tests at the University of Florida several years ago [Walton et al, 1999; 2004]. In these tests various depths of 3*mm* glass beads were placed in a 4*cm* diameter acrylic pipe with a slightly smaller diameter cylindrical pedestal forming the base of the glass-bead bed.

The pipe was pulled up in an Instron universal test apparatus and the force required to move the pipe recorded as a function of the bed height. This setup replicates the assumptions of Janssen's silo analysis and the force on the bottom pedestal approached the Janssen asymptotic value when the bed height was a few pipe diameters. The coefficient of friction and the ratio of axial to radial stress were not measured in these tests, so that the product μk became a fitting parameter for the Janssen prediction of vertical force.

It is worth noting that the assumptions in Janssen's silo stress-distribution analysis do not include any assumption or statement that the granular material is at its maximum stress, or is in a state that is on or near its failure surface. The only assumption regarding 'failure' is the assumption that the wall friction is fully-developed (i.e., at the sliding limit). Generally Coulomb wall-friction only places an upper limit on the sliding wall-friction force (and it is possible that the wall-friction force could be less than the sliding limit). If there is some small relative displacement between the wall and the material however (e.g., due to settling of the material, or due to any motion of the wall itself) then the wall-friction is likely to be at, or very near, the sliding limit. Particle-scale simulations with spherical particles have shown that (depending on the ratio of wall friction to interparticle friction coefficients) rotation of spheres near the wall may affect whether the stress distribution assumed by Janssen is obtained [Chester, et al, 2009].

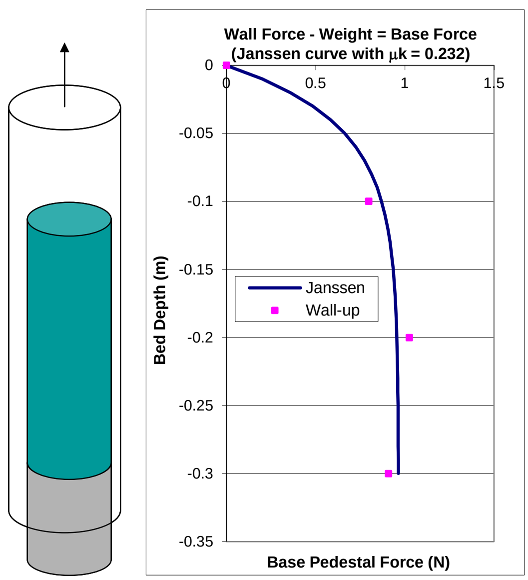


Figure 2. Force on base pedestal as wall is pulled up as a function of bed height. Symbols – 3mm glass beads in 4cm acrylic pipe, Line – Equation (2) [from: Walton, 2004; 1999]

Some researchers [e.g., Rotter, 2008; Brookfield, 2015] interpreting Janssen's stress distribution assume that the granular material inside the column is at or near its failure limit, which for a non-cohesive granular solid with an internal friction angle, φ, would have the maximum and minimum principal stresses, σ_1 and σ_3, in the material at failure, related by

$$\sigma_3 = \sigma_1\,[(1 - \sin\varphi\,)/(1 + \sin\varphi)] \qquad (3)$$

and the assumption is often made that the factor, *k*, in Janssen's analysis should be given by the factor in the square brackets in Eqn (3); Various expressions are used to infer a value for *k*, usually based on shear-strength measurements of the granular material (e.g. from shear-cell tests). The civil engineering standards for silo design in Europe often use an estimate for $k \approx 1.2$ (1 - sin φ), where φ is the measured internal friction angle for the granular solid (i.e., at failure) [Schultze, 2006]. However, as mentioned above, nowhere in the assumptions of Janssen's stress analysis is there any criterion, or assumption about the granular material being at, or near, failure. The only assumption regarding the stress response of the granular material is that the material behaves somewhat like most linear elastic solids (e.g., with a constant Poisson's ratio). For an isotropic material at stress states below its failure limit, the stress ratio, *k*, of Janssen's analysis might be more-closely related to a Poisson's ratio (i.e., which could range from a value near 0.2, up to as high as 0.45 for a material like sand, depending on its state of compaction).

Measurement of Floor-Pressure under Dry and Submerged Sand & Gravel Columns

Laboratory-scale tests were devised that would determine whether pipe-wall friction supports a significant fraction of the total weight of sand and gravel poured into the top of relatively tall vertical pipes. The apparatus selected was suitable for both dry and submerged conditions. Unlike the Fig 2 glass-bead test, previously performed at the University of Florida, a direct measurement of floor pressure was desired, instead of a measurement of the load transferred to a pipe-wall which was translated vertically before or during the measurement. Previous particle-scale computational studies of floor-stresses in granular columns have encountered some difficulty producing results consistent with Janssen's theory without at least some vertical motion of the pipe wall [e.g., Chester, 2009]. In the current study some preliminary tests which were attempted using a load-cell to measure the load transmitted through a semi-flexible bottom membrane. These tests demonstrated that care needed to be taken to ensure that the pipe-wall load does not affect the measurement of the floor-pressure (as described below, and in Appendix A).

The floor-pressure measurement method selected for the dry and submerged granular-column study utilized a thin latex membrane to seal the bottom of vertical 3-inch (7.6cm) diameter PVC pipes (and 6-inch PVC pipes), with a water-filled chamber below the latex membrane. A simple manometer was used to manually measure the pressure in the water-filled chamber below the granular column. The diameter of the manometer tube was selected to be as small as practicable (~2mm-dia) in order to minimize the deflection of the latex membrane. During the tests it was determined that the small size of the manometer tube led to a small hysteresis in the pressure measurement, due to capillary effects, depending on whether the pressure increased or decreased before the reading (i.e., pressure uncertainties of around ±3mm H_2O, or ±30Pa, were observed). The membrane between the vertical PVC pipe and the water-filled chamber below the membrane was sealed and held in place via a Van Stone PVC flange attached to the bottom of the vertical PVC pipe, as shown in Figure 3. Appendix A describes some initial attempts to measure floor-pressure using a separately mounted load-cell or digital scale below a pipe 'rigidly' mounted to a frame or wall. In those initial attempts the compliance of the pipe-mount allowed small vertical displacement (sagging) of the vertical pipe, which confounded the floor-load measurement with the sensitive load-cell or digital scale. The manometer attached directly to the bottom of the pipe containing the granular material eliminated any relative motion between the measurement system and the pipe, and produced much more consistent and reliable results. Appendix A also includes a figure showing floor load measurements by Widisinghe [2014] which looks suspiciously like it suffers from the same confounding of the response to mounting-compliance with floor-load of the granular material inside the model-silo that we encountered in our initial attempts to perform such measurements.

The very first floor-pressure test performed with this apparatus was a 'submerged-test' wherein the PVC pipe was first filled with water (to a height of ~193cm) and a 'lift' of ~700g of sand was poured through the water; then subsequent 'lifts' of ~700g of aquarium-gravel (~4- 5mm dia) were poured through the water to fill the pipe. This test resulted in unexpectedly low floor-pressure values (described in detail later). The unexpectedly low floor-pressure values prompted a systematic evaluation of floor-pressure loads under both dry and submerged conditions in both 3-in and 6-in (7.6cm and 15.2cm) diameter pipes, including the short columns with weight added on the top surface, used to verify some of the new theoretical relations developed.

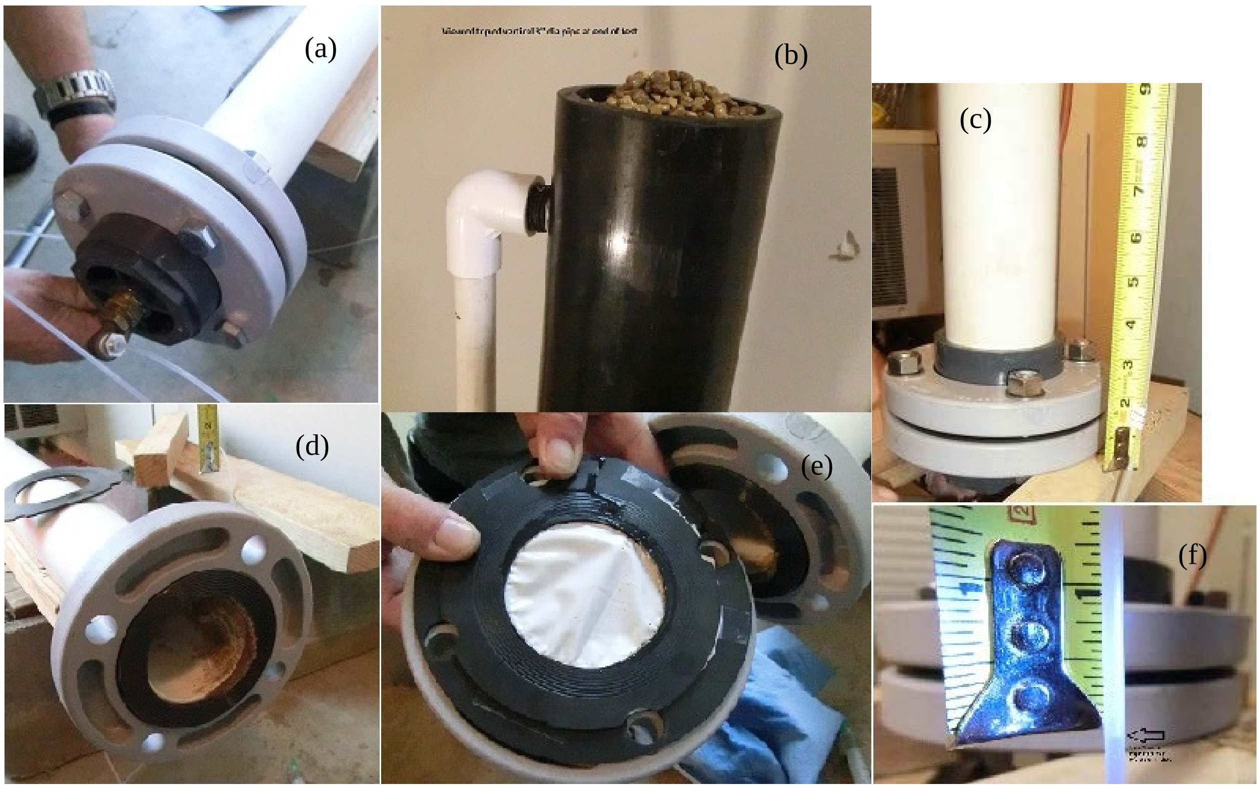


Figure 3. (a - f) show images of the 3-inch (7.6cm) diameter PVC pipe and Van Stone flanges holding a thin latex diaphragm separating the sand column from a water-filled chamber below the pipe. (a & f) The water-filled chamber is connected to 2mm-diameter manometer tube, where (f) shows the water level in the manometer tube (e.g., arrow) at the beginning of test (before adding sand, gravel or water to the PVC pipe). (b) shows aquarium-gravel filling the pipe (a short section of ABS pipe was used at the top of the column).

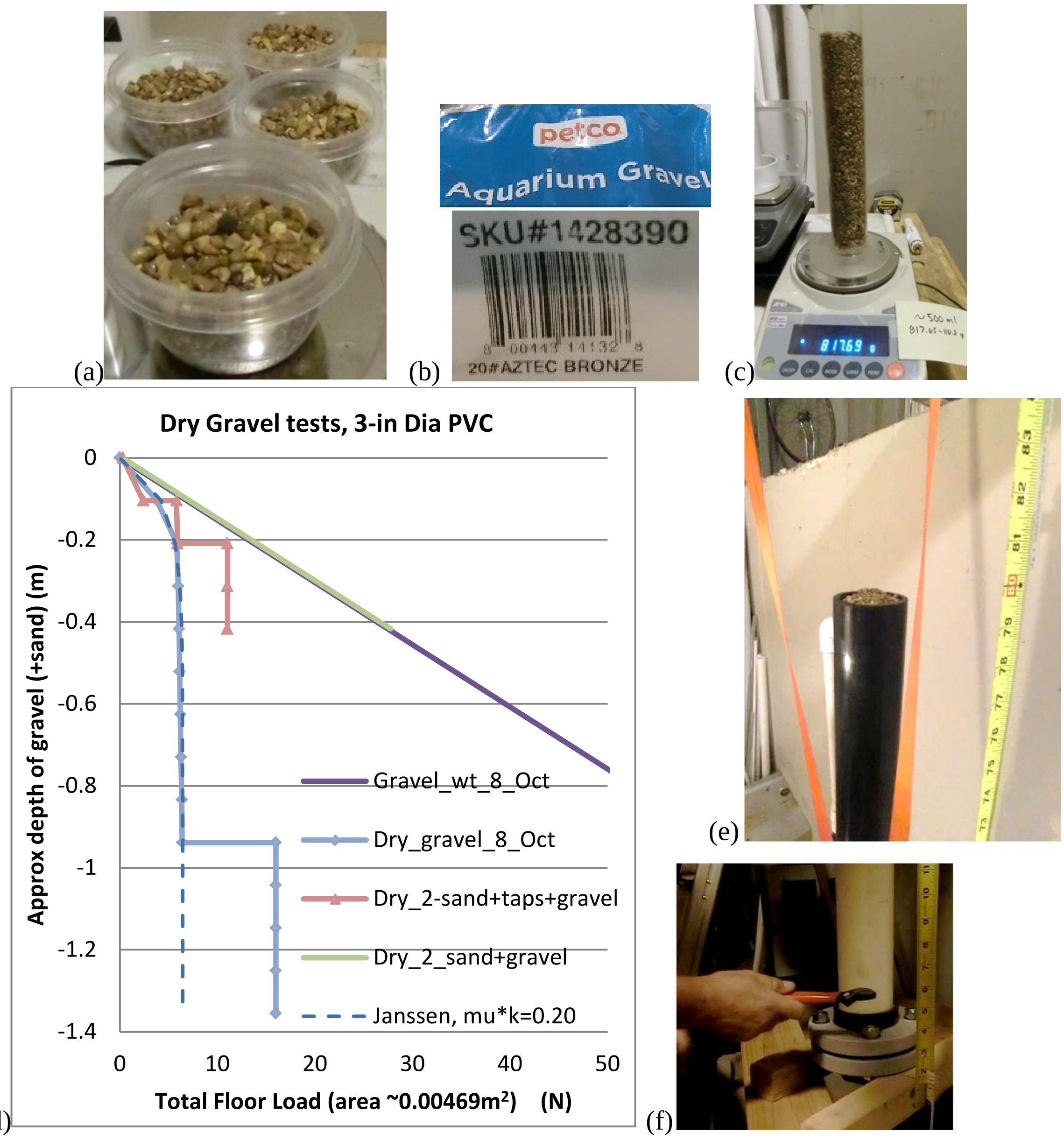
In the 3-inch diameter tests the first ~1.3-diameters of 'fill' material in the bottom of the vertical pipe was sand (#30 sieve Monterey beach sand, which appeared to be primarily quartz). Bulk density measurement of the sand 'as-poured' into a 500ml graduated cylinder was ρ_b ~1.49g/cc. Upon tapping (i.e., with a graduated cylinder manually held down on top of a Gilson vibrator in 'tapping-only' mode) the bulk density increased by nearly 6% to ρ_b ~1.58g/cc). During the submerged floor-pressure tests a mass of ~256.5g of water was displaced when 700g of sand was added, indicating a grain density ρ_g ~2.73g/cc for the sand. Thus, these dry bulk-density measurements indicate that the porosity of the sand as-poured in the graduated cylinder is around 45%, and it decreases to around 42% with mild tapping (~200 'taps' of the Gilson vibrator). Interestingly, when this same sand was used in the 6-inch PVC pipe studies (filling through the same funnel, but with some radial flow after hitting the bed) the bulk-density of the dry sand was significantly lower, ρ_b ~1.37g/cc, indicating an initial porosity close to $\phi_{6\text{-}inch} \approx 50\%$ instead of the $\phi_{2\text{-}inch} \approx 45\%$ as occurred in the smaller diameter graduated cylinder. Yet, in submerged 6-inch tests the initial porosity of the sand (after falling through a column of water to form the sand-bed in the bottom of the pipe) had a porosity $\phi_{submerged} \approx 47\%$. As described later, this variability in initial density of the sand at the bottom of the granular columns probably was a major contributing factor in the variability seen in some of the resulting measurements.

Measurement of Floor-Load in 3-in (7.6-cm) Diameter Dry Gravel over Sand Column

Figure 4 shows the results from a dry test in the 3-inch diameter pipe of Fig 3. The light-blue solid curve (diamonds) is the floor load measured in a test in which 700g of sand were initially poured into the top of an approximately 2m tall vertical pipe, and successive 700g 'lifts' of gravel were added on top of the sand. The dashed blue line following the same general curve as the initial part of the floor-load test data, is a Janssen curve (Eqn 2) fit to the data using a single value of $\mu k = 0.2$ for both the sand and gravel portions of the curve. The horizontal portion of the measured floor-load curve shows the behavior of the floor-load when the pipe-wall was manually 'tapped' repeatedly, at several heights and around the circumference (by hitting it manually with a wench as shown in Fig 4f). Interestingly, the floor load would both increase and decrease as the tapping proceeded. The decreases in floor load were most noticeable when the tapping moved to a different height (especially to a lower height after tapping at a higher level). We hypothesized that the tapping may have increased the solids-packing in the gravel in a region where tapping is occurring (as well as temporarily reducing the wall friction at that height). Material at higher packing might exhibit a higher *k*-value, and subsequent tapping below that level might leave some material 'held-up' by a compacted arch or high *k*-value region so that its load is not transmitted lower down. This explanation is somewhat speculative since no systematic investigation was performed to determine why the floor-load sometimes decreased in response to tapping; however, it is a plausible explanation for the observed behavior. The vertical portion of the measured floor-load curve is the unchanging floor-load that occurred when several additional lifts of gravel was added to the pipe after the wall-tapping had taken place. The second test curve with the solid reddish line (triangles) showed a lower floor load after the sand-lift was introduced (it was not noticed at the time of this test that the load was lower than in the previous test). If a Janssen curve were fit to that single data point it would require a somewhat higher value of μk than was used to fit the first loading curve. After the sand-lift was placed in the pipe, it was tapped until no further increase in floor load was observed with additional tapping (several seconds of tapping around the circumference were needed to reach this limiting value). Addition of the next lift of gravel had no effect on the floor load, but the floor load could be made to increase by tapping on the pipe wall (temporarily reducing the wall friction). Additional lifts of gravel added after the second tapping sequence produced no increase in the measured floor-load.

Simple measurements of bulk properties were used to roughly characterize the sand and gravel. Assuming the bulk density of the sand in the pipe was roughly the same as in the 500ml graduated cylinder, the depth of the sand with one 700g 'lift' was roughly 10cm, or ~1.3 pipe-diameters. Additional 'lifts' of Petco Aztec-Bronze aquarium gravel (dia < ~6mm) were added to the pipe, and the floor pressure recorded after each 'lift'. The aquarium gravel had a bulk density of ~1.42g/cc in a 500ml graduated cylinder (and the bulk density obtained in the ~2m tall, 3-in diameter pipe was ~1.43g/cc). In later submerged tests the water displaced was measured for each lift of gravel added, providing information for a reasonable estimate of the grain density of the aquarium gravel ($\rho_{g\text{-}gravel} \approx 2.55$g/cc). This allowed an estimate of the gravel porosity to be obtained, $\phi_{gravel} \approx 0.44$.

(a) (b) (c)



(d) (e) (f)

Figure 4. (a-c) Images of 4-6mm diameter Aquarium Gravel. (d)Floor-load from dry sand plus aquarium-gravel in a vertical 3-inch PVC (and an ABS extension) pipe. The upper straight lines are the total weight added. The (light-blue) solid line is the measured floor load and the dashed line is a Janssen curve (Eqn 2 with μk = 0.2) fit to the data. The horizontal portions on the measured curves show the response of the floor-load to tapping on the pipe wall (i.e. lightly hitting the pipe wall multiple times around it circumference and at different heights with a wrench as shown in the video frame on the lower right-hand side. (e) 3-in PVC (& ABS) vertical pipe. (f) 'tapping' pipe with wrench.

Submerged Granular Columns (buoyancy correction for Janssen's analysis)

In order to correctly account for the effect of interstitial water and the hydraulic head of water in a submerged floor load configuration, the definition of the *density*, ρ, of the granular material in the column used in the earlier differential-slice analysis leading to Eqn 1 needs to be defined more precisely. In Janssen's analysis the density refers to the *bulk density*, ρ_b, of the granular material in the bed, including the void volume. Usual terminology for granular solids defines the porosity, ϕ, as the fraction of the total volume, V_{tot}, occupied by the void space, V_{void},

$$\phi = \frac{V_{void}}{V_{tot}} \tag{4}$$

The mass of solids in a granular bed, M_s, with total volume, V_{tot}, is given by,

$$M_s = V_{tot}(1 - \phi)\rho_g \tag{5}$$

Where, ρ_g, is the grain-density of the solid particles making up the granular material. This expression assumes that all porosity is *connected* interstitial space, or, if it is interior to the particles, it is accessible for air or fluid to penetrate into and out of freely (i.e., there is no closed or inaccessible porosity – or if there is any closed porosity, it is considered to be part of the *grain-density* of the material*)*.

For the submerged systems considered here we assume a fixed *head* of fluid, h_f, exists above the floor, and its height is unaffected by the addition of the solids to the granular column. In the tests described here, this constant *head* of fluid was accomplished by having an overflow pipe at a height, h_f, above the floor, or in a few cases without an overflow pipe, the overlying water-level was adjusted to its original level, by removing some water after each 'lift' of solid was added. When solid material is added in a submerged configuration, a buoyancy correction (accounting for the mass of fluid displaced) needs to be made to determine the effective mass of the added solids, M_{eff}, given by,

$$M_{eff} = V_{tot}(1 - \phi)(\rho_g - \rho_f) \qquad \text{(for } \rho_g > \rho_f\text{)}$$

In the earlier differential slice force-balance analysis, the gravitational weight term, $\rho g \pi R^2 dy$, then, has a buoyancy-corrected form,

$$\textit{(dry)} \quad \rho g \pi R^2 dy \quad \Rightarrow \quad \textit{(buoyancy corrected)} \quad g(1 - \phi)(\rho_g - \rho_f)\, \pi R^2 dy$$

The floor-pressure for the submerged case needs to use this modified density term and also include a term for the fixed hydraulic head. The resulting buoyancy-adjusted Janssen-like expression for the floor pressure, σ_y, (i.e., a submerged form of Eqn 2) then is,

$$\sigma_y = g\rho_f h_f + \frac{g(1-\phi)(\rho_g - \rho_f)R}{2\mu k}\left[1 - e^{-\frac{2\mu k}{R}y}\right] \qquad \text{(for } \rho_g > \rho_f\text{)} \tag{6}$$

Where, for tests where the pipe wall is opaque and no explicit measurement is made of the incremental bed height, y, it can be estimated from the incrementally measured weight of material added, W, the final height, y_{max}, and the total weight corresponding to the maximum height, W_{max},

$$y \approx \frac{W}{W_{max}}\, y_{max}$$

For weight added to the top of a submerged granular column (e.g. a top-surface stress of σ_0) the form of the submerged floor-load expression becomes,

$$\sigma_y = \underbrace{g\rho_f h_f}_{\textit{Hydraulic head}} + \underbrace{\frac{g(1-\phi)(\rho_g - \rho_f)R}{2\mu k}\left[1 - e^{-\frac{2\mu k}{R}y}\right]}_{\textit{Load from bottom layer}} + \underbrace{\sigma_0 e^{-\frac{2\mu k}{R}y}}_{\textit{top surface load}} \qquad \text{(for } \rho_g > \rho_f\text{)} \tag{7}$$

from which we obtain an expression for expected floor-loads from submerged granular columns consisting of two (or more) layers, with the bottom layer 1 having a height of y_1,

$$\sigma_y = \underbrace{g\rho_f h_f}_{\textit{Hydraulic head}} + \begin{cases} \frac{g(1-\phi_1)(\rho_{1g}-\rho_f)R}{2\mu_1 k_1}\left(1 - e^{-\frac{2\mu_1 k_1}{R}y}\right), & for \quad y < y_1 \\ \underbrace{\frac{g(1-\phi_1)(\rho_{1g}-\rho_f)R}{2\mu_1 k_1}\left(1 - e^{-\frac{2\mu_1 k_1}{R}y_1}\right)}_{\textit{Load from bottom layer}} + \underbrace{\frac{g(1-\phi_2)(\rho_{2g}-\rho_f)R}{2\mu_2 k_2}\left(1 - e^{-\frac{2\mu_2 k_2}{R}(y-y_1)}\right)}_{\textit{Load from upper layer}} \underbrace{e^{-\frac{2\mu_1 k_1}{R}y_1}}_{\textit{Reduction factor}} & for \quad y \geq y_1 \end{cases} \tag{8}$$

Measurement of Floor-Loads in 3-in (7.6-cm) Diameter Submerged Gravel over Sand Columns

Figure 5 shows the floor load vs height measured in submerged tests using the same apparatus and materials as in Fig 4. The two submerged tests were run on different days and the quantity of water in the manometer differed slightly (leading to a systematic shift of 0.22N between the two days 'measurements' – the overflow pipe was at the same height but the starting 'head' due to water filled to the overflow height, produced a reading of 88.18N on the Oct-1 test and 88.4N for the Oct-8 test. This systematic shift in pressure of ~5mm-H_2O in the manometer reading was not considered significant). The floor-load measured in the Oct-1 test (dashed reddish line) was much lower than anticipated (discussed below). The solid blue-green line (diamonds) in Fig 5 can be compared to a prediction based on the single-material form of Eqn-6 (blue dashes) using the same value of μk = 0.2 as was used to fit the dry measured floor load in Fig 4. A 2-layer model was also fit to the submerged test data, and the dotted green line in Fig 5 shows the result of using Eqn 8 with a value of μk = 0.3 for the submerged sand and μk = 0.088 for the submerged gravel. As can be seen, the 2-layer functional form (i.e., Eqn 8) can be made to fit the measured curve somewhat better than the single term form of Eqn 6. No independent measurements were made of k and μ under submerged conditions, so the product μk was used as a fitting-parameter to see whether the functional form of the derived equations could reproduce the shapes of the measured curves.

In the dry floor-load tests in Fig 4 the lift of sand placed in the pipe for the second test produced a noticeably lower floor load than had occurred with one sand-lift of 700g (~10cm sand height) in the first test. In the *submerged* tests shown in Fig 5 the differences in the floor-loads between two successive tests were significantly greater than in the earlier dry tests. The dashed (dark-red) curve (circles) in Fig 5 shows the measured floor-load vs depth from the first (i.e. Oct-1) *submerged* 3-inch diameter test. In this test only 0.365N was added to the floor load when 4.35N of weight (700g, buoyancy corrected) was added to the water column. In the subsequent *submerged* test (Oct-8) using the same type of sand and the same apparatus, the floor load increased by just over 2N when the first 700g of sand was added to the pipe (buoyancy corrected weight of 4.32N). The difference in floor-load from these two nearly identical operations, with the same equipment, was approximately <u>a factor of 5.5.</u> At the time the Oct-1 test was performed, we did not notice that the initial slope of the floor-load curve appeared significantly different than the buoyancy-corrected weight curve (i.e., corresponding to $(1 - \phi)(\rho_g - \rho_f)gh$), since the results were not being graphed in real time. At the end of that test, however, we were a little surprised by how little the floor-load had increased. The manometer tube was checked, post-test, to verify that it was functioning satisfactorily (e.g. the tube was manually raised and lowered to verify that the head remained at the same height – the previously mentioned ±2mm, or so, hysteresis was noted, but nothing unusual was found). Nothing out of the ordinary was detected.

A glance at the graph of the floor-load measurement from that first submerged test (dashed red curve, circles Fig 5) makes it fairly obvious that this test produced unusual floor-load behavior. The differential-slice analyses of Janssen (e.g. Eqn 2) and other modifications for different boundary conditions, all produce floor-load vs height curves that have an initial slope equal to $g\rho_{eff}$, which is the same as would occur in a liquid or the 'lithostatic' slope in saturated geologic beds. Typical silo floor-load vs depth curves only begin to deviate from lithostatic behavior at finite distances below the top surface. Although we did not measure floor loads with very fine height resolution, the average slope over the first ~1.3-Diameter lift-height differs dramatically from the slope of a lithostatic head (i.e., the 'buoyancy-corrected' weight line appearing very close to the top of Fig 5). We also found that it was not possible to select a single material characteristic μk value that would produce a Janssen-like curve (i.e., Eqn 6) that behaved like the measured floor load in the Oct-1 test. It was also not possible to construct a 2-layer (i.e., Eqn 8) curve with the first layer having a thickness corresponding to the full height of the sand (i.e., one-lift of 700g, ~10cm height in the 3-in pipe). If a μk value were selected that would produce a Janssen curve (for the 1st lift) and also go through the measured point at ~10cm, the slope of the floor load from any material added on top of that material would be so steep that the floor load would hardly increase at all as the gravel was added. The only way that a two-layer (e.g. Eqn 8) curve could be made somewhat similar to the measured curve was to assume that a relatively thin layer with a very-large value for $\mu_1 k_1$ occurred at the very bottom of the sand, and that the material above that behaved in a manner similar to the materials in the other submerged tests. The 'dotted' light-blue line was obtained by assuming a 2cm thick layer with $\mu_1 k_1$ = 1.6 to be at the bottom of the sand, and that the material above that layer behaved as if it had $\mu_2 k_2$ = 0.088. An even better fit to the measurement could be obtained if a thinner layer (i.e., ~1cm thick) with a higher $\mu_1 k_1$ = 4 value were beneath an upper layer with $\mu_2 k_2$ = 0.049 (producing the dotted purple line in Fig 5). We are not suggesting that these are the only interpretations of the material properties in the various layers in the test that could explain the data, but they certainly suggest plausible reasons for part of the observed floor-load behavior, especially if the characteristics of over-consolidated sand are considered.

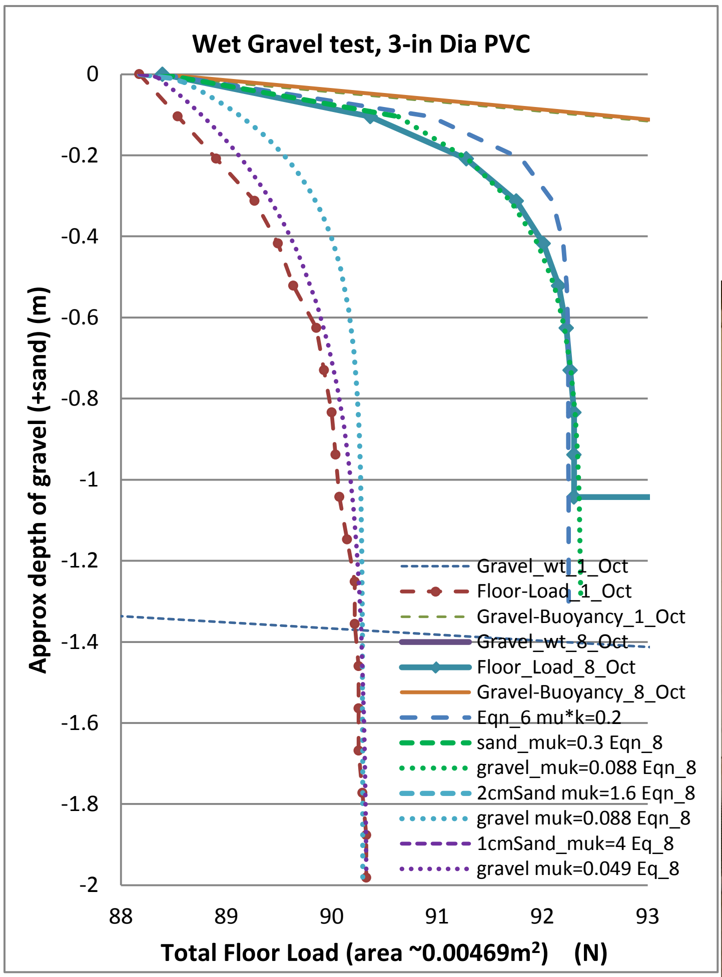


Figure 5 Results of 3-in (7.6cm) diameter submerged floor-load tests. The solid blue line (diamonds) is for the Oct-8 test. The dashed line just above it is a single-material submerged Janssen curve (Eqn 6 with μk = 0.2). The dotted green curve following the Oct-8 measurement is a 2-layer (Eqn 8) curve with $\mu_1 k_1$ = 0.3 and $\mu_2 k_2$ = 0.088 depth of 10cm, corresponding to the approximate sand depth. Lines near the dashed-red Oct-1 measured curve are attempted fits to that curve, using Eqn 8. The blue-dotted line is Eqn 8 with $\mu_1 k_1$=1.6 for a depth of 2cm then $\mu_2 k_2$=0.088 above that; the purple dotted line is with $\mu_1 k_1$=4 for a depth of 1cm, then $\mu_2 k_2$=0.04 above that.

A well-known characteristic of saturated beach-sand, which has been deposited under submerged conditions, is that it is often in an *overconsolidated* state, such that it dilates in response to almost any deformational loads (as described by Reynolds [1885]). Anyone who has ever walked or run on the saturated sand remaining behind as waves flow out, knows that that particular region of sand resists deformation (i.e., appears significantly stronger) than most dry sand on the beach. Quantitative studies of sand clearly show that if it has been consolidated (through vibration, loading, or because of the manner in which it is deposited) so that its void-fraction is less than a 'critical' value, it is considered to be *overconsolidated*; and in response to any shearing deformation, it will dilate, or it can generate large dilation stresses resisting the shear deformation. Although we did not perform any specific tests of sand consolidation, or on the detailed condition of the sand as a function of height in the vertical columns, it seems entirely plausible that some of the sand deposited by falling through the 2-m column of water could have formed a layer which is overconsolidated. It also seems reasonable to ascribe higher *k*-values to such material. That being said, we do not have any quantitative measurements to base the material properties we are ascribing to the thin-strong-layer in these curve-fits to the floor-load measurements. When the 3-inch diameter submerged floor-load test was repeated a week later (e.g., the Oct-8 curve of Fig 5), the floor-load vs depth measurements could be fit quite well via straightforward application of the one-layer or two-layer analytic expressions (e.g. Eqn 6 or Eqn 8). We do not have an explanation for the differences between these two tests, other than possible small differences in the manual pouring of sand or gravel into the top of the vertical water columns.

Comparison of Submerged and Dry 3-in (7.6cm) Diameter Granular-Column Floor-Loads

Figure 6 shows the measurements from the dry and submerged 3-inch PVC pipe tests on the same figure for ease of comparison between the two cases. The measured floor load from the dry (sand plus gravel) test had an asymptotic floor-load value of 6.38N which less than the load (6.86N) in the first lift of sand added to the pipe which created a bed with a depth of ~1.3 Diameters (e.g., 700g of sand). In other words the maximum dry floor load (in filling the 0.076m diameter column to over 7-diameters in height) never exceeded the load from a depth of 1.3-D of unconfined material.

When looking at the submerged data it is immediately clear that the sand and gravel added to the ~25-D high water column created a very small increase in the total floor load. The maximum measured increase in floor load was ~3.9N (or about 90% of the buoyancy corrected load, 4.33N, added to the column, when the first 700g lift of sand was added). Again in this case it can be said that the measured floor load never exceeded the (buoyancy corrected) load from a depth of 1.3-D of unconfined material (which also corresponds to the lithostatic load from a depth of about 0.8D of dry material, i.e. not buoyancy corrected).

Floor-Load in ~2m Tall, 15.25-cm Diameter Submerged Granular Column of Gravel over Sand

Figure 7 shows various views of the apparatus and materials used in a ~2-meter tall (two-layer) submerged floor-load test. Fig 7a shows the latex diaphragm over the Van Stone flange which fit on the bottom of a 2.1m tall ~6in (~15.25cm) diameter transparent PVC pipe (Figs 7e and f). Fig 7b shows the PVC reducer on the lower Van Stone flange which formed the constant volume water chamber connected to the manometer tube (not shown). The same #30 mesh Monterey beach sand as was used in previous tests was used as the first layer in this test; however the gravel in the second layer was changed to a material with a wider size distribution (e.g. up to ~12mm or so) and with more angular particles than the rounded Aquarium gravel used in the previous tests. Fig 7g shows the granular column after 9kg of sand has been poured through the 2m high water column. No leveling of the sand layer was done in this test. Fig 7h shows the top portion of the granular column after gravel was added up to a few centimeters below the water exit/overflow port. The gravel as purchased (from the garden department at Home Depot) looked reasonably clean (as shown in Fig 7d); however, as seen in Fig 7h, after pouring the gravel into the water column, the water was very murky with silt. After the gravel was added to the pipe, fine silt could be observed through the transparent pipe wall, slowly flowing down through the interstitial space and depositing on the top side of gravel throughout the bed. After the test the unit was left in place with the water and gravel inside overnight. The next day the silt had settled out of the water (leaving a clear head ~10cm deep above the gravel bed) and deposited a silt layer that looked like frosting covering the top surface of the gravel in the pipe. While this silt was quite noticeable in suspension, it occupied only a small fraction of the total interstitial space in the gravel and is not believed to have had any significant effect on the mechanical behavior of the gravel column.

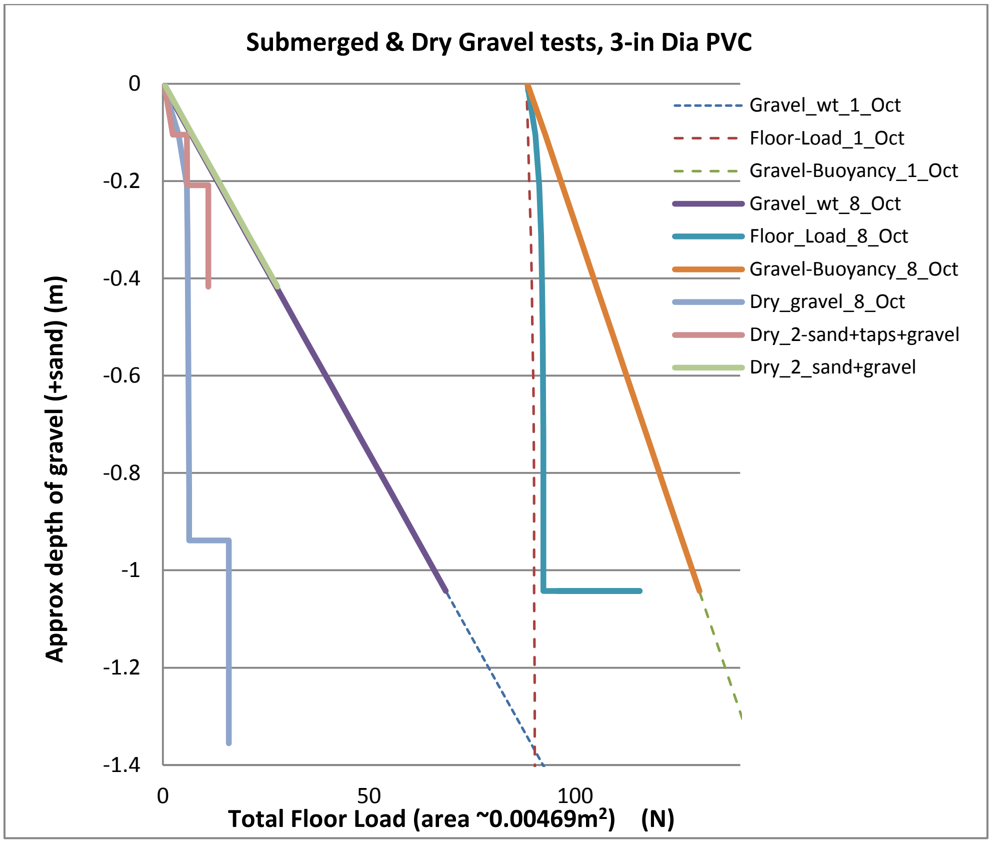


Figure 6 The experimental curves from Figs 4 and 5 on the same scale showing that the combination of the buoyancy-correction and the material-property changes in going from dry to submerged conditions reduces the floor-load increase that occurs when sand and gravel are added to a vertical cylindrical pipe. This would imply that a dry floor load test would be used to generate a conservative (i.e., worst-case) estimate of the 'additional load' for submerged conditions.

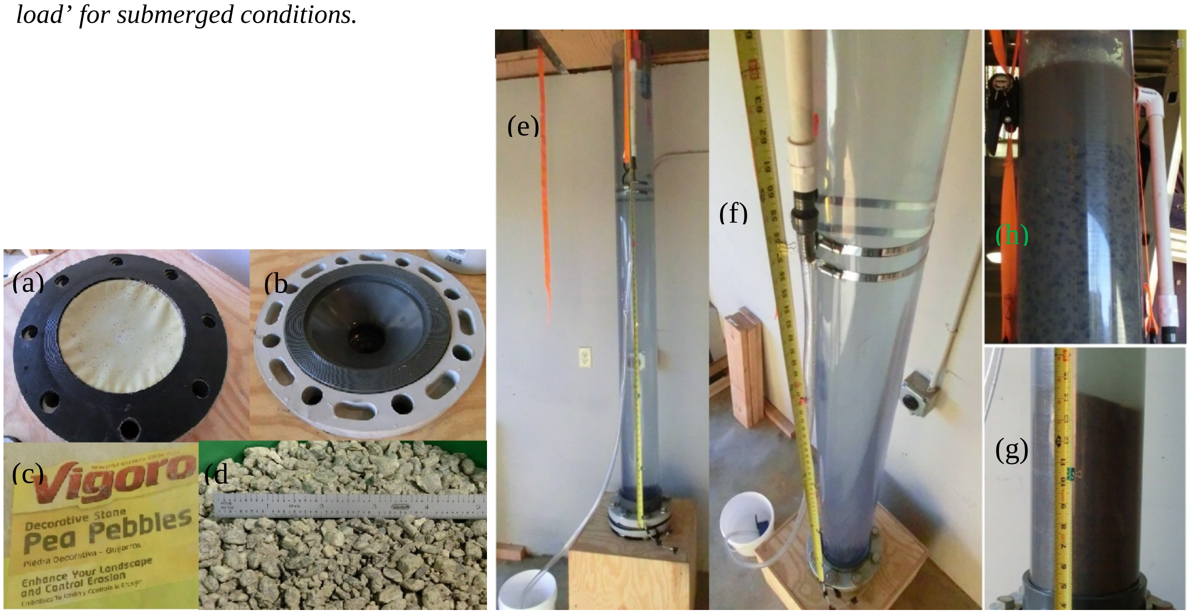


Figure 7. The 6-inch (~15.25cm) diameter, 2.1-m tall PVC pipe, Van Stone flange, latex diaphragm, gravel and images of sand and gravel columns in the pipe during the submerged tests. Individual images are described in the text.

Figure 8 shows the floor load measured (solid line with squares and diamonds) beneath the submerged 6in (~15.25 cm) diameter × 2.1m tall sand and gravel column. The submerged Janssen curve (Eqn 7 or Eqn 8) fitting the sand test data under the water head was nearly the same as the value for μk used to fit floor load data for dry sand tests (e.g., $\mu k \approx 0.14$).

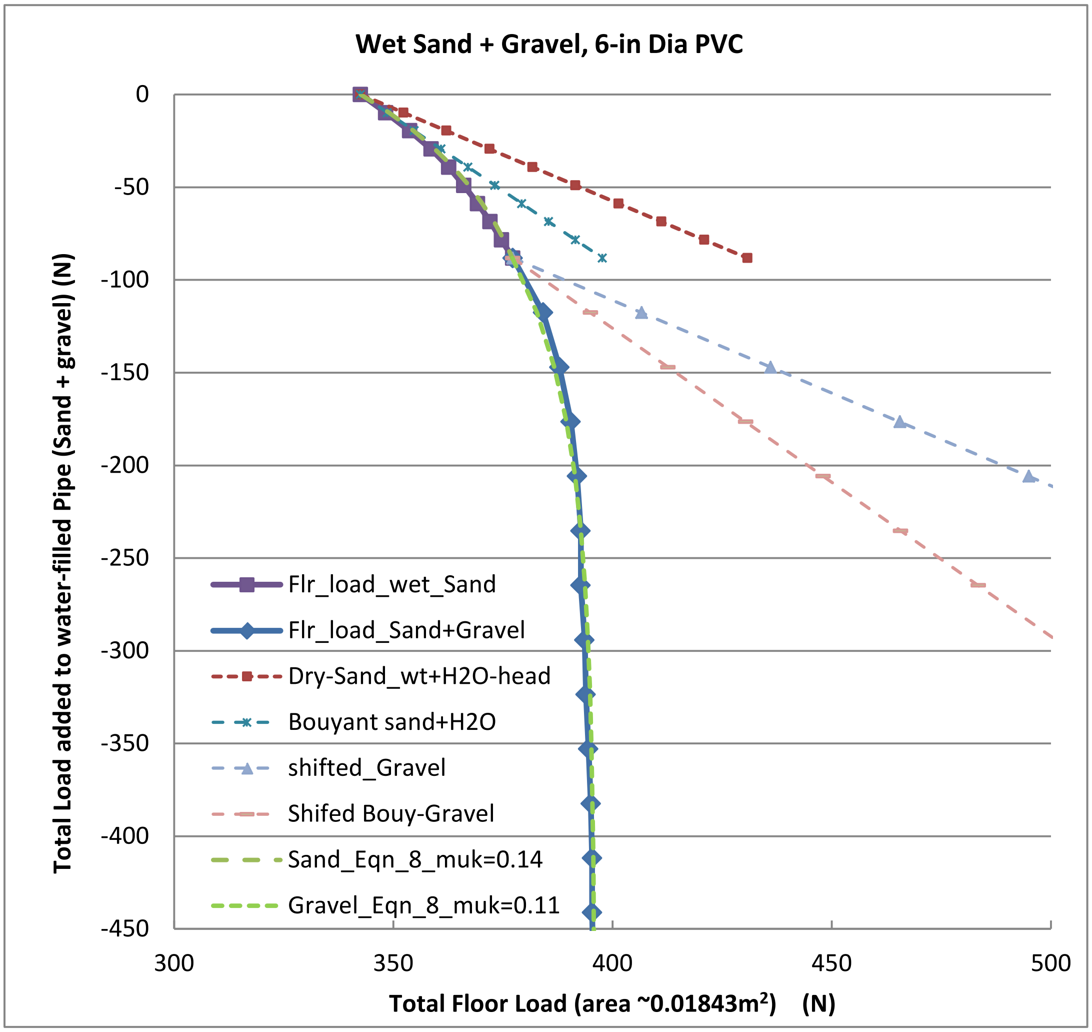


*Figure 8 Floor load measured (solid line with squares and diamonds) beneath the submerged 6in (~15.25 cm) diameter × 2.1m tall sand and gravel columns. The uncorrected weight of the sand (dashed curve with red squares) the buoyancy-corrected sand weight (dashed blue curve with *'s), the uncorrected gravel weight (dashed curve with triangles) and the buoyancy corrected gravel weight (dashed light-red curve with horizontal bars) represent loads that would develop at the bottom of an unconfined bed (i.e., without pipe-wall friction).*

The addition of gravel (Fig 7 c, d and h) on top of the initial sand bed (Fig 7g) produced floor-loads (solid line with diamonds in Fig 8) that were consistent with a submerged, layered Janssen curve (Eqn 8) using $\mu_2 k_2 = 0.11$ for the gravel. Also shown on Figure 8 are curves for the uncorrected weight of the sand (dashed curve with red squares) the buoyancy-corrected sand weight (dashed blue-green curve with *'s), the uncorrected gravel weight (dashed light-blue curve with triangles) and the buoyancy corrected gravel weight (dashed light-red curve with horizontal bars). As can be seen the maximum 'extra' floor-load over that from the water head produced by the sand and gravel together in the column, never exceeded the buoyancy-corrected load that a height of 2.3 diameters of unconfined sand would have produced (dashed light-blue curve with *'s) and was considerably less that the load that would have been produced by a depth of 2.3 diameters of unconfined dry sand (dashed curve with red squares). No other tests were performed using this gravel.

Discussion/Conclusions –
Floor-loads from submerged and dry vertical columns of gravel over sand were measured and in general found to be consistent with an analysis based on Janssen's differential slice load-balance approach for determining the floor-loads in silos. Submerged tests with 1 to 2 diameters of sand covered by a taller layer of relatively fine gravel in 3-inch and 6-inch dry and submerged PVC pipes all show a load that asymptotes after a height of a few diameters. The sand in the larger diameter dry tests appeared to be deposited with a somewhat higher porosity than in smaller diameter dry configurations or than under submerged conditions. One 3-inch pipe test under submerged conditions exhibited unusually low floor loads which could not be reasonably explained using only the layered differential slice analysis. We hypothesize that in that test a relatively thin region of submerged sand may have been deposited in an overconsolidated state (like sand on the beach) and thus exhibited high shear-dilatent forces which produced high wall friction. There are other possible explanations, none of which fully explain the exceptionally low loads in that one test.

In general the asymptotic extra floor-load produced by adding sand and gravel to a water filled column, produced less additional floor-load than the lithostatic pressure under a height of sand or gravel corresponding to 2 pipe diameters (and usually less than the added load produced by a buoyancy-corrected unconfined bed of submerged sand or gravel 2-diameters in height). The PVC pipes used in these tests were not modified in any way to change their inherent surface friction. Thus, they had smooth relatively low friction inner walls. Real uncased boreholes would be expected to have higher wall friction coefficients than the values μ that developed in the PVC pipes, and would thus have lower floor-loads than those obtained in this study.

Acknowledgements –
This work performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344. The source Physics Experiment (SPE) Project at LLNL supported this work under the supervision of Drs Tarabay Antoun and William Walter. That support and their encouragement is gratefully acknowledged.

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Appendix A- Potential Confounding of Vertical Wall-Loads with Floor-Loads in Lab Tests

The utilization of a manometer attached to the bottom of the model granular column shown in the figures in the body of the text was not the first configuration considered for the measurement of floor-loads. Two earlier configurations were attempted before settling on this configuration. The initial configurations mounted the vertical pipe to a rigid wall, placed a flexible seal over the bottom of the vertical pipe. In this configuration the seal material needed to be strong enough to hold the hydrostatic head of the water column anticipated in the vertical pipe. For this purpose a commercially available clamp-on rubber cap was placed over the end of a vertical PVC pipe (and the interior of the PVC pipe was covered with a rough adhesive-backed sandpaper to provide a high friction wall). A flat plate just slightly smaller in diameter than the inner pipe diameter was placed below the rubber cap, and a load cell was centered below the plate to measure the load delivered to the plate. The load cell mount was elevated just slightly so that a positive 'zero' load existed when there was no material or water in the pipe. Both dry and submerged measurements of floor load vs material added to the pipe were conducted. The results initially looked very similar to a theoretical Janssen-like curve; however as the total load in the pipe increased instead of asymptoting to a constant value the measurements indicated a nearly linear increase in floor load with weight of added material. The behavior was observed for both submerged and dry tests. Later a 2nd smaller set up was constructed to do a more detailed study of the effects observed. During the tests with the smaller pipe dry tests were conducted with a gap of a few grains thickness between the bottom of an open pipe and a container sitting on a digital scale (i.e., a load cell). It was observed that sometimes the load measured by the scale would increase and/or decrease in unexpectedly large amounts – this was found to be due to sand grains wedging between the bottom of the pipe and the digital-scale-supported open container. It was also observed that, after an initial load vs depth curve that looked like a Janssen prediction, a linear change in load with material-added appeared to occur (albeit at a much smaller slope than in the initial tests with a 6-inch diameter pipe). A careful review of the measurement setup and wall-mounting procedures used in these tests showed that the load-cells were very sensitive to small vertical displacements of the pipe. The wall-mounts (of the pipes to the rigid walls) had enough elastic compliance to allow the weight held up by the pipe to cause the mount to 'sag' slightly. This 'sagging' of the wall mounts (or mount 'compliance') caused the load cells to receive a portion of the wall load which was supposed to be held entirely by the wall mounts. The load was transmitted to the load cell through compression of the rubber cap in the initial submerged-tests with a 6-inch diameter pipe, and through wedged sand-grains between the pipe bottom and the load-cell-mounted container in the dry tests with a small gap below the vertical pipe.

A cursory review of online literature shows other experimental measurements of floor loads in model silos which may well have had wall loads and floor loads confounded (e.g. Widisinghe and Sivakugan, [2014]). Figure A1 shows the experimental results of one of their measurements compared to a Janssen-like analysis of the same configuration.

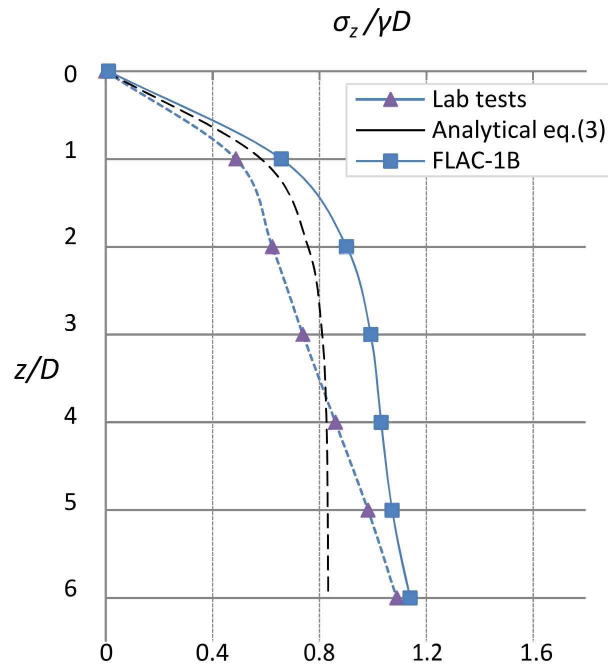

Figure A1 (from Widisinghe, 2014) showing lab tests of floor load from sand in a model silo where the vertical pipe was suspended above a digital scale with a "one-grain diameter" gap between the suspended pipe and the floor-load measuring digital scale. The nearly linear loading with additional material is very similar to the results obtained in this work, before we accounted for compliance of the pipe supports and ensured that movement of the wall did not affect the floor pressure measurement.