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Title: Multi-Channel Correlation Detectors: Accounting for and Reducing Non-target Detections

Author(s): Carmichael, Joshua Daniel

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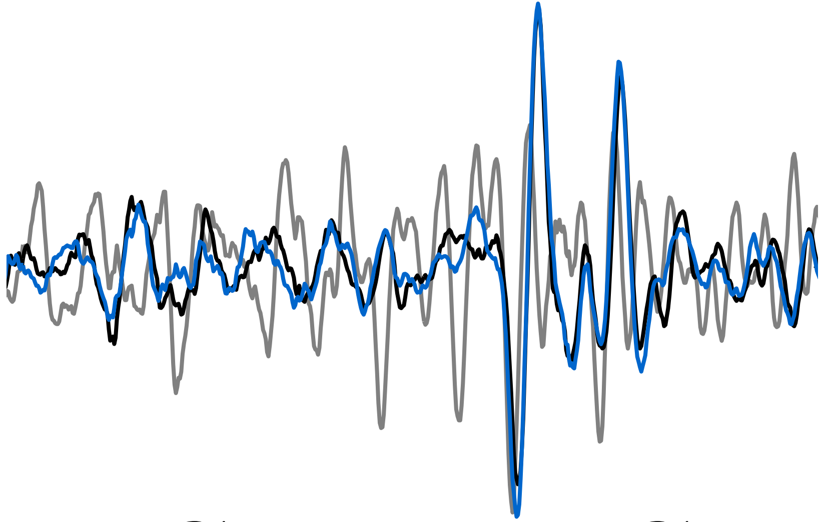


Multi-Channel Correlation Detectors:

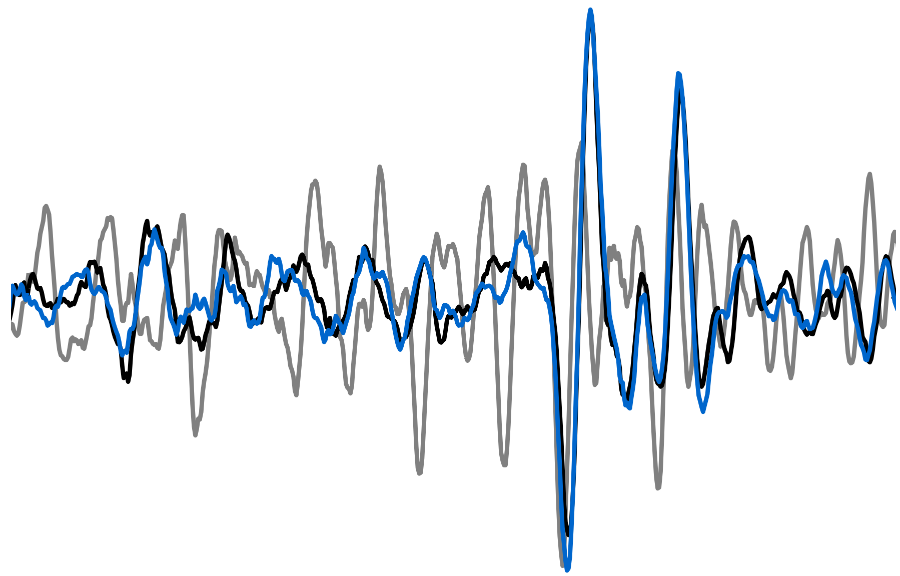
Accounting for and Reducing Non-Target Detections

Joshua D Carmichael
EES-17, Geophysics

General Detection Goal

Recognize Signal from Source Despite Noise/Interference

Depends on prior knowledge of the anticipated Source Signal and Noise Environment



Depends on volume of data from anticipated Source Signal and Noise

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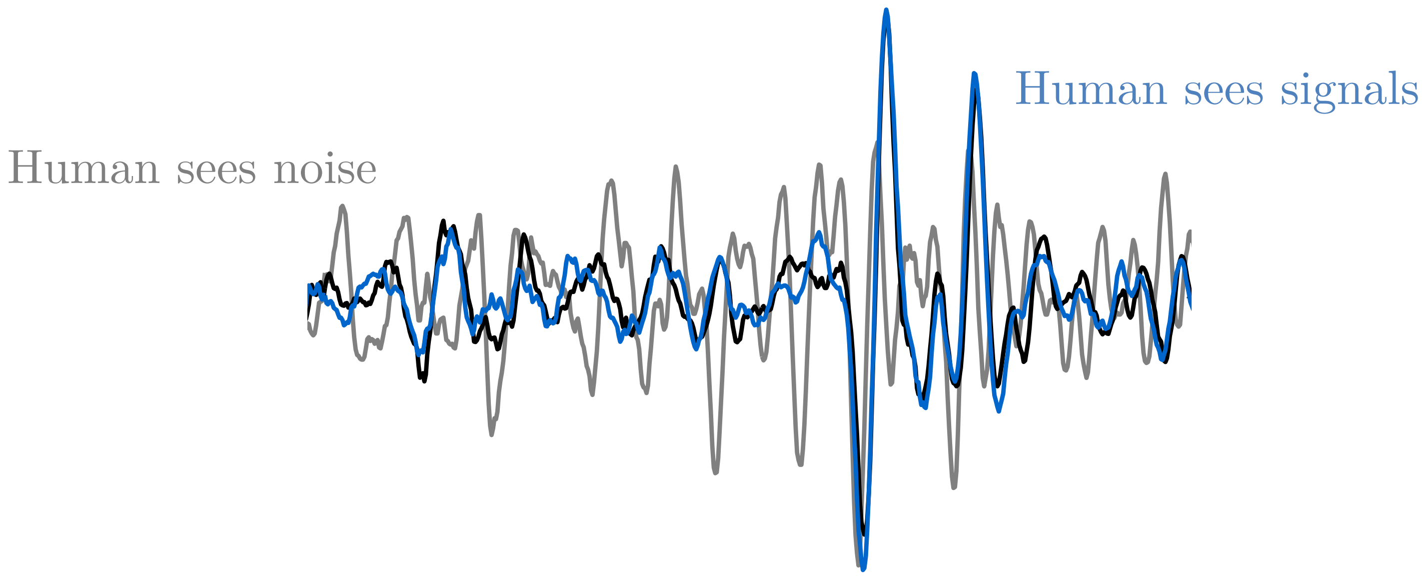
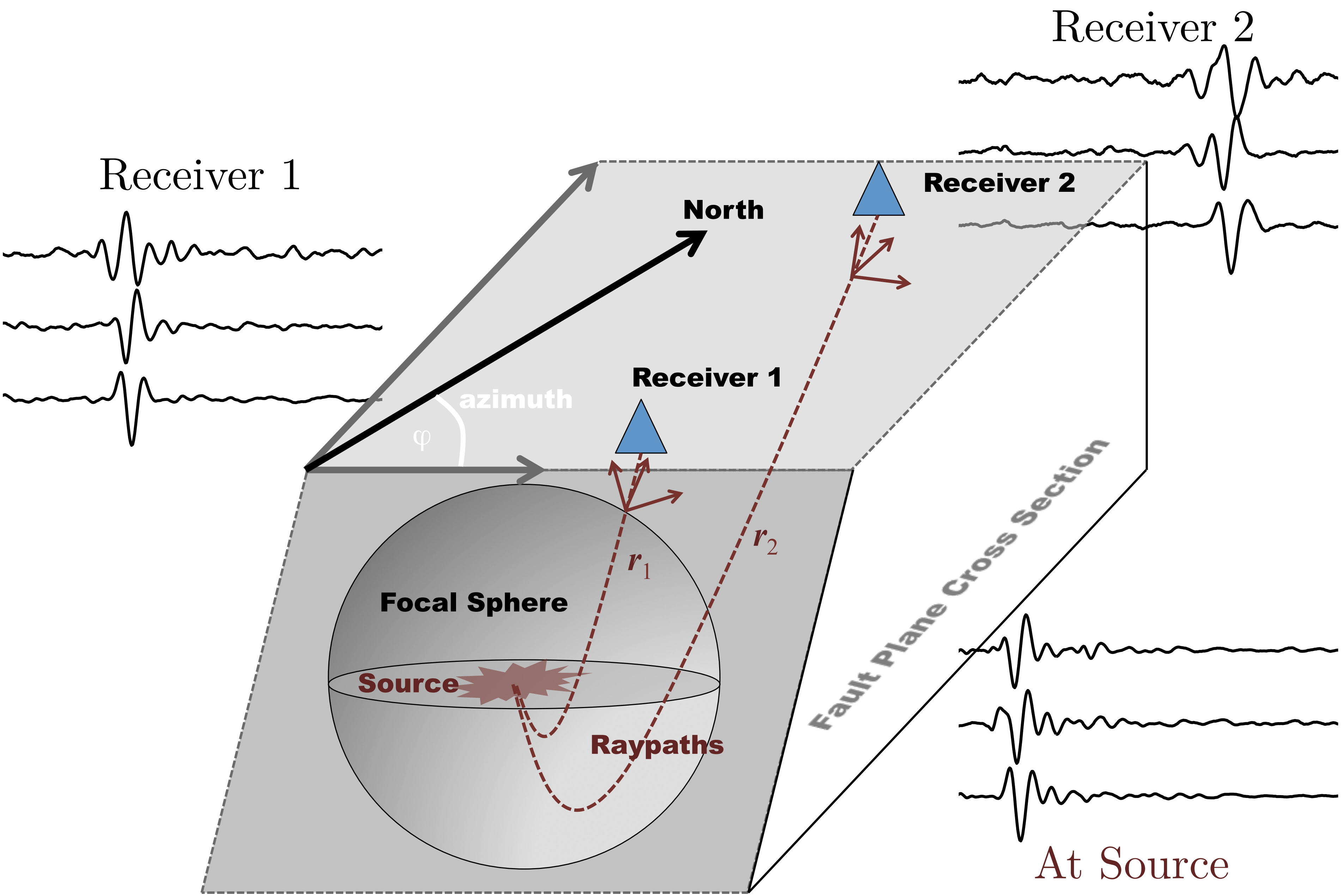


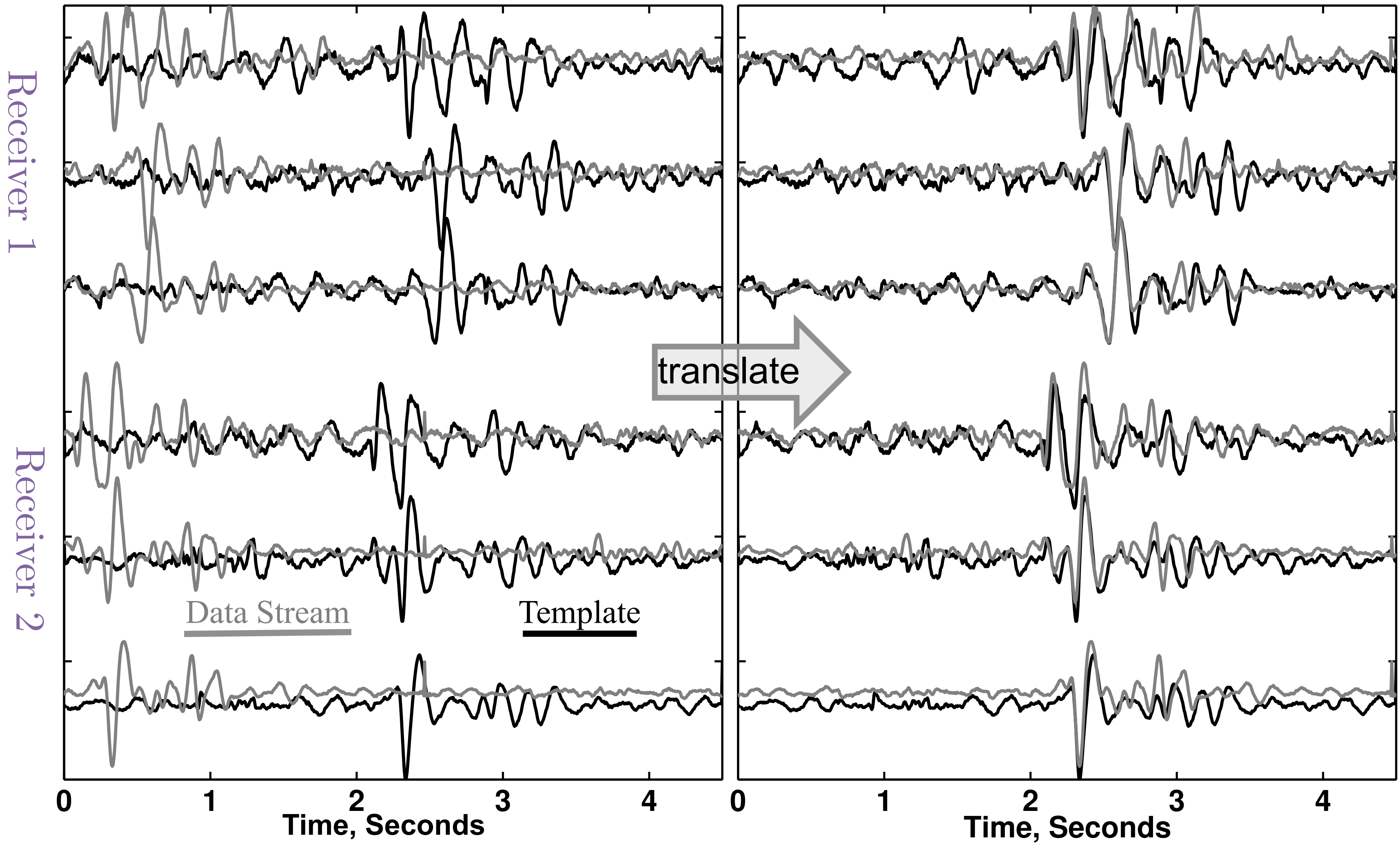
Depends on volume of data from anticipated Source Signal and Noise

GNDD Multichannel Signals



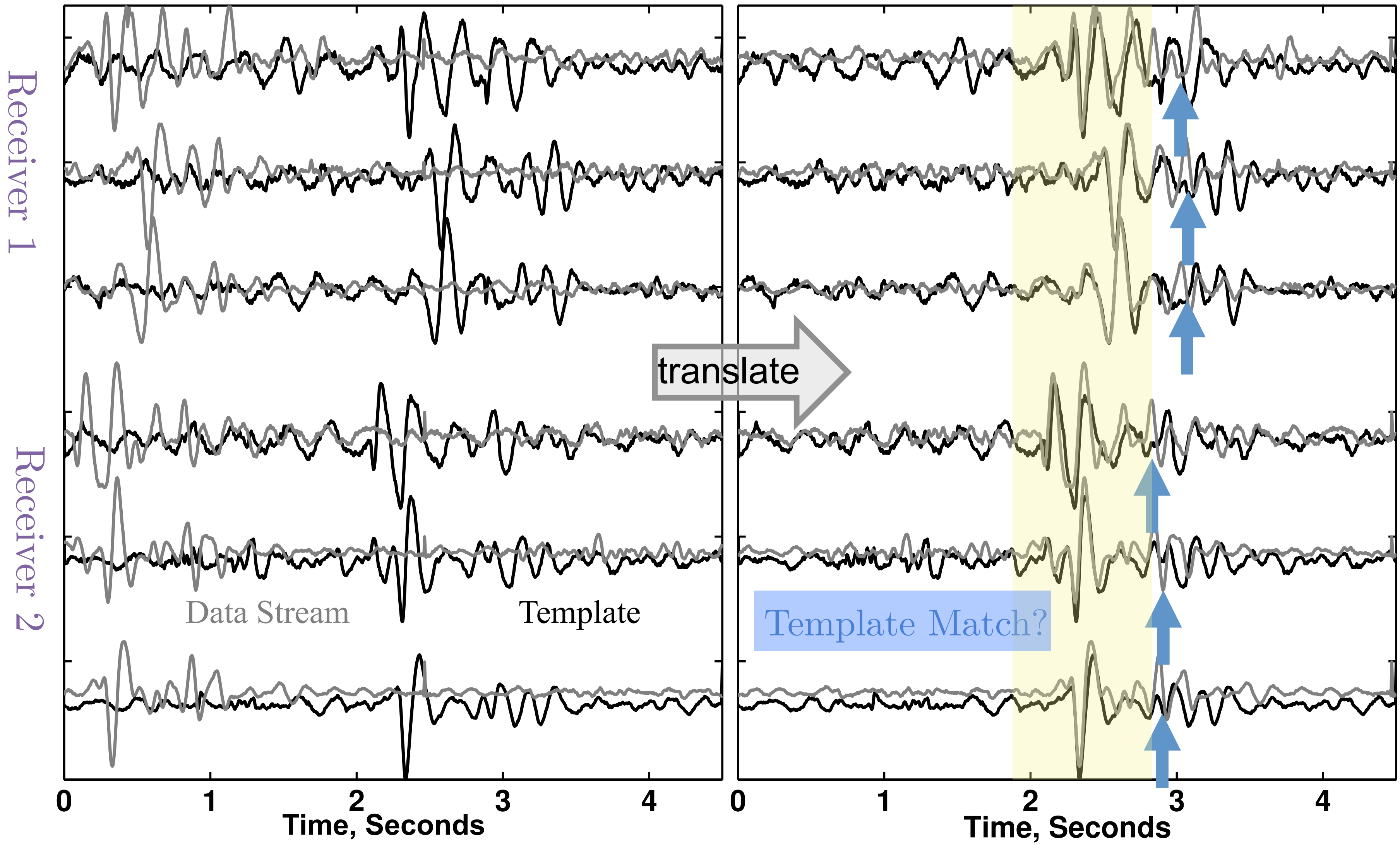
Correlation Detector Implementation

Scan Data with Template to Search for Repeating Event



Correlation Detector Implementation

Evaluate if Estimated Correlation Value in Detection Window indicates Repeater



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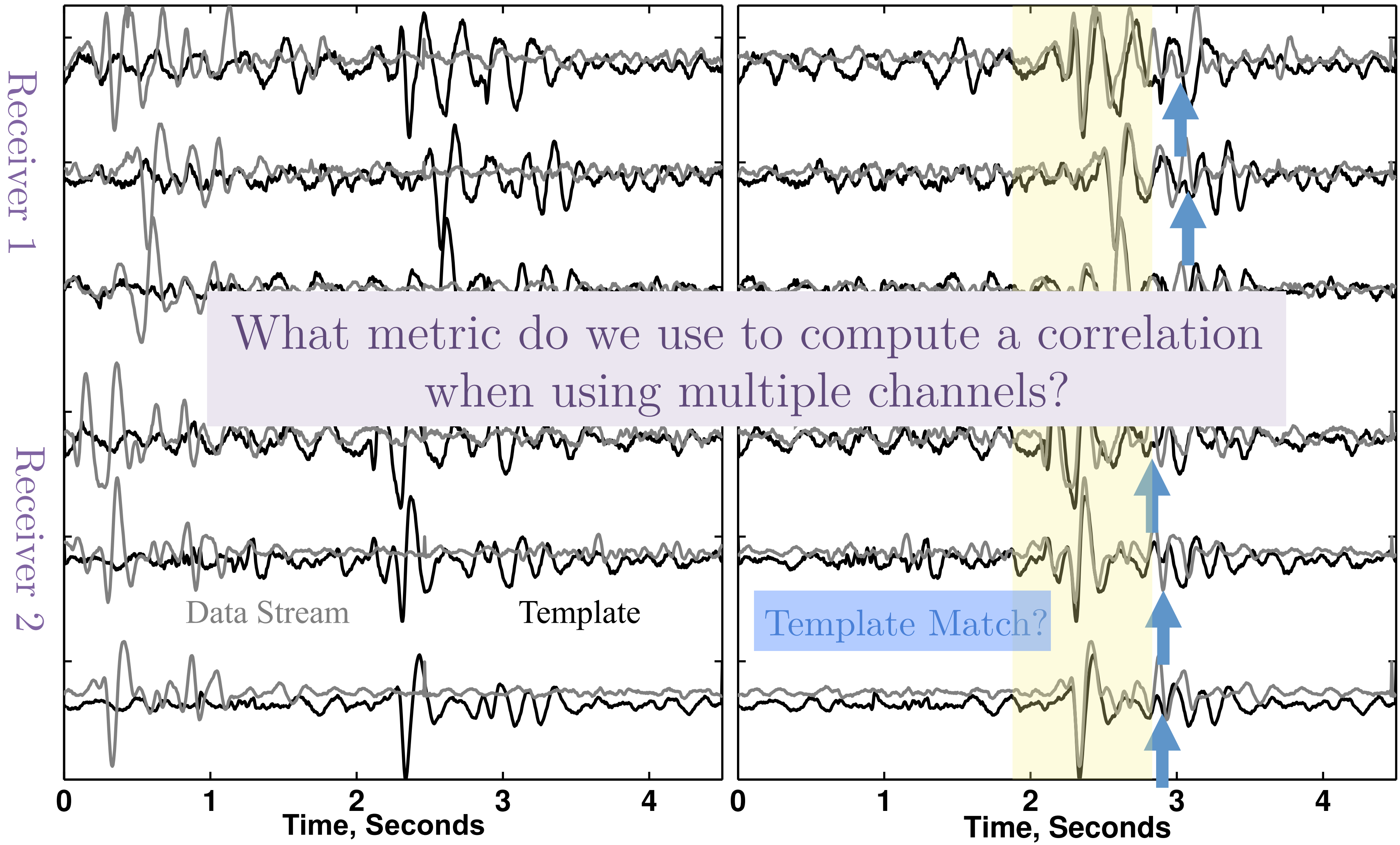
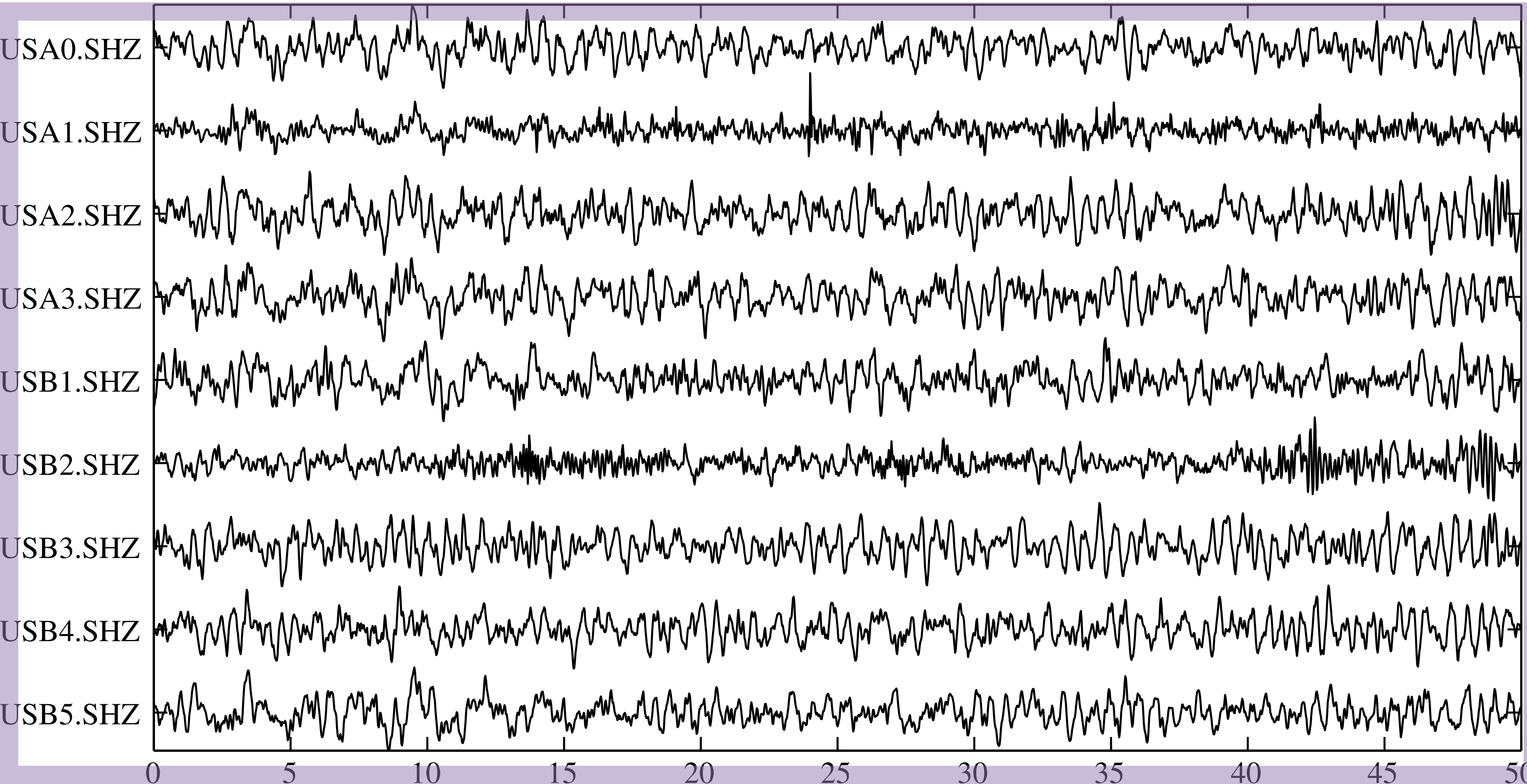


What metric do we use to compute a correlation when using multiple channels?

Correlation Detector Hypotheses

$$\mathcal{H}_0 : \quad \boldsymbol{x} = \boldsymbol{n}_0 \quad \text{(noise only)}$$

$$\mathcal{H}_1 : \quad \boldsymbol{x} = \boldsymbol{n}_1 + A\boldsymbol{u} \quad \text{(noise, plus waveform pulse)}$$



Multi-Channel Correlation Statistic

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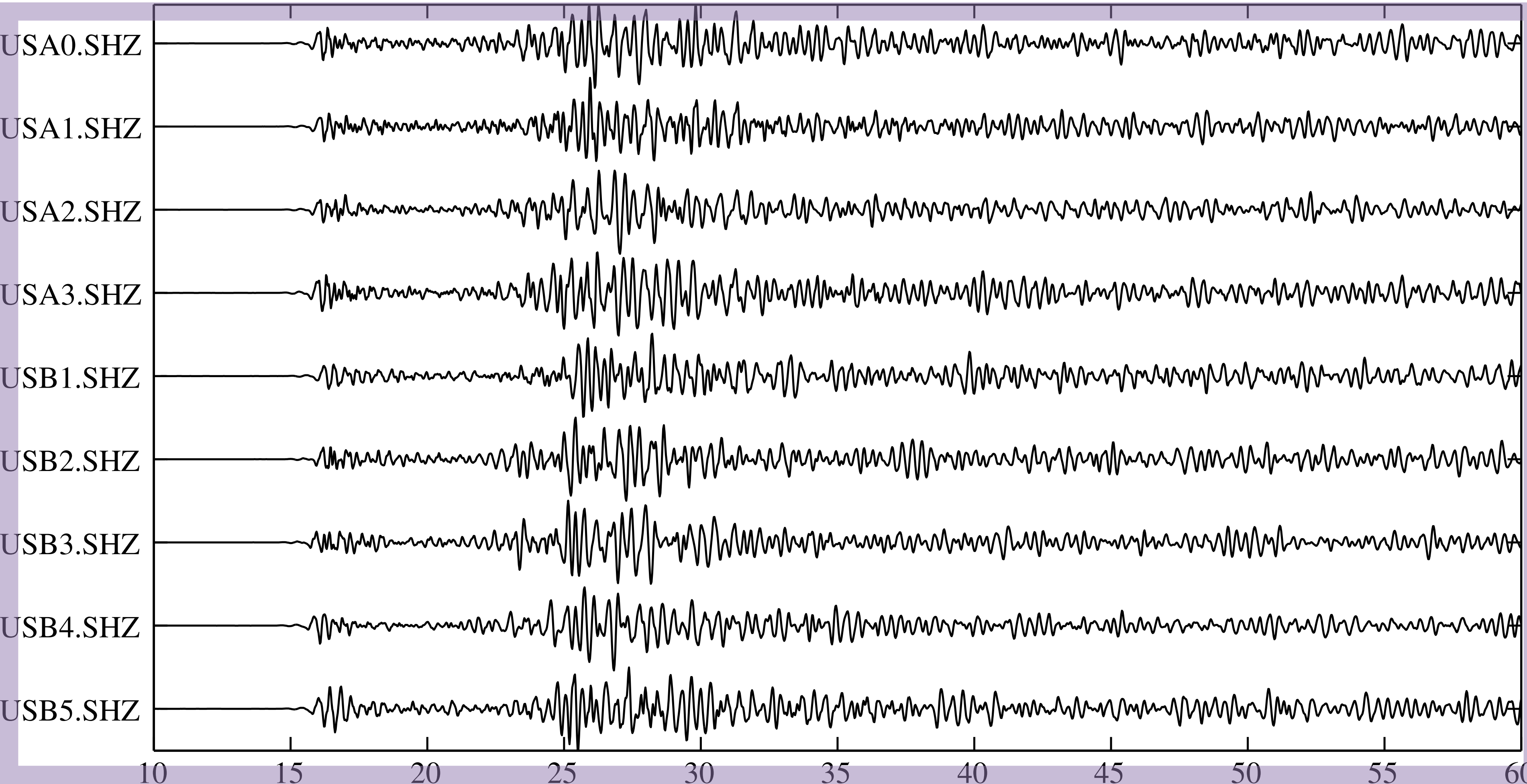

Multichannel Detection Challenges

Questions—ordered by difficulty

1. What is the best way to combine single channel correlations? Can it be demonstrated?
2. What if the template waveform is uncertain, or the target data originates from a much smaller source?
3. What if the ambient wavefield isn't composed of noise alone (it's not)?

Respective Solutions

1. Beam provides higher detection capability for r than MLE, *at moderate correlation values.*
2. Quantitative analysis: nuisance alarm rate increases *dramatically* for template-target match degradation
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Multichannel Detection Challenges

Questions—ordered by difficulty

1. What is the best way to combine single channel correlations?

This isn't a Trivial Question!

Practical Consequence:

Different Detection Statistics for the Same Detector yield different results: over time, this can amount to $\sim 10^3$ missed or false detections

moderate correlation values.

2. Quantitative analysis: nuisance alarm rate increases *dramatically* for template-target match degradation
3. Make a detector more specific than a correlation detector by modifying the null

MLE Correlation Statistic

Q: What is the best way to combine single channel correlations?

Signal models and hypotheses

$$\mathcal{H}_0: \quad \boldsymbol{x} = \boldsymbol{n}_0 \sim \mathcal{N}\left(\boldsymbol{0}, \sigma^2 \boldsymbol{I}\right)$$

$$\mathcal{H}_1: \quad \boldsymbol{x} = \boldsymbol{n}_1 + A\boldsymbol{u} \sim \mathcal{N}\left(A\boldsymbol{u}, \sigma^2 \boldsymbol{I}\right)$$

Detection Statistic from Generalized Likelihood Ratio

$$s(\boldsymbol{x}) = \frac{\max_{A,\, \sigma_1}\{ p_1(\boldsymbol{x};\, \mathcal{H}_1) \}}{\max_{\sigma_0}\{ p_0(\boldsymbol{x};\, \mathcal{H}_0) \}}$$

Decision Rule: Is there a signal match on multiple channels?

$$s(\boldsymbol{x}) = \frac{\langle \boldsymbol{x}, \boldsymbol{u} \rangle_F}{||\boldsymbol{u}||_F ||\boldsymbol{x}||_F} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

Harris Statistic

MLE Correlation Statistic

Q: What is the best way to combine single channel correlations?

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Detection Statistic from Generalized Likelihood Ratio

$$s\left(\boldsymbol{x}\right) = \frac{\max_{A,\sigma_1}\{p_1\left(\boldsymbol{x};\mathcal{H}_1\right)\}}{\max_{\sigma_0}\{\ldots\}}$$

Neyman-Pearson CFAR Constraint

$$\text{Const} = \Pr_{FA} = \int_{\eta}^{1} p_0\left(s\,;\,\mathcal{H}_0\right) ds$$

Decision Rule: Is there a s

$$s\left(\boldsymbol{x}\right) = \frac{\langle \boldsymbol{x}, \boldsymbol{u}\rangle_F}{||\boldsymbol{u}||_F ||\boldsymbol{x}||_F} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

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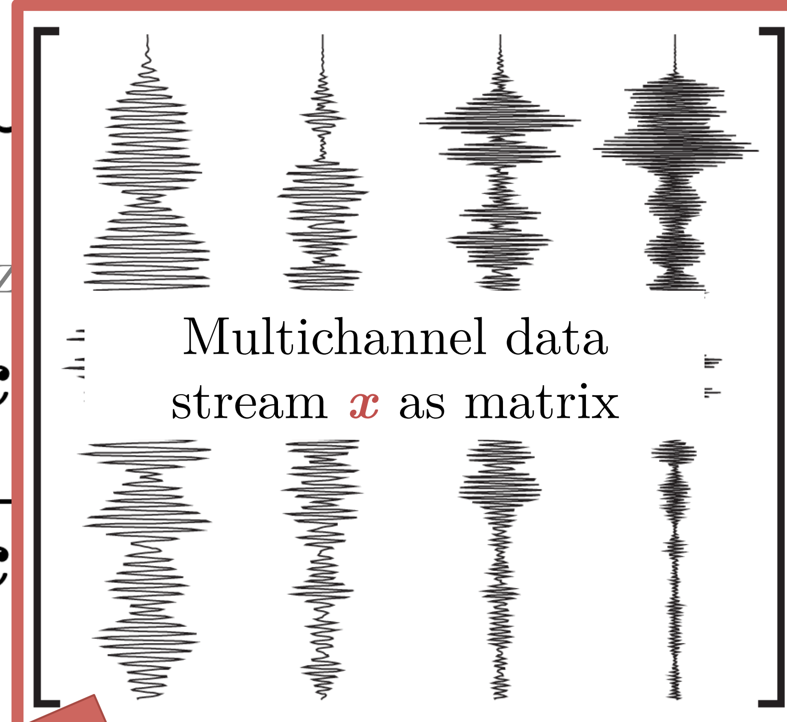
Detection Statistic from Generaliz

$$s\left(\boldsymbol{x}\right) = \frac{\max_{A,\, \sigma_1}\{ p_1 \left(\boldsymbol{x}\right.}{\max_{\sigma_0}\{ p_0 \left(\boldsymbol{x}\right.}$$



$$\langle \boldsymbol{x}, \boldsymbol{u} \rangle_F = \mathrm{tr}\left(\boldsymbol{x}^{\mathrm{T}} \boldsymbol{u}\right)$$

$$s\left(\boldsymbol{x}\right) = \frac{\langle \boldsymbol{x}, \boldsymbol{u} \rangle_F}{||\boldsymbol{u}||_F ||\boldsymbol{x}||_F} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

Harris Statistic

Computation: The Correlation Detector

Q: What is the best way to combine single channel correlations?

Practical point 1: Multi-Channel data can be organized into matrix columns, or multiplexed into long vectors.

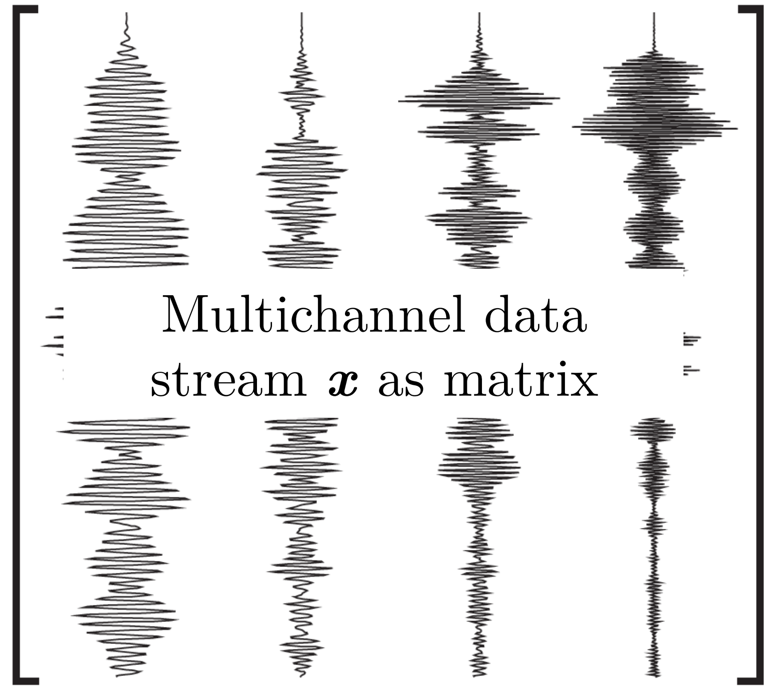
Practical point 2: No reason data covariance **C** is diagonal.

We use a reduced Degree of Freedom Estimator to correctly parameterize PDF for $s(x)$, despite $\mathbf{C} \neq \mathbf{I}$

$$\boldsymbol{x}(t) = \left[\ \boldsymbol{x}_{11}(t),\quad \boldsymbol{x}_{12}(t),\quad \dots\quad \boldsymbol{x}_{N3}(t)\ \right]$$

$$\boldsymbol{u}(t) = \left[\ \boldsymbol{u}_{11}(t),\quad \boldsymbol{u}_{12}(t),\quad \dots\quad \boldsymbol{u}_{N3}(t)\ \right]$$

$$\boldsymbol{n}(t) = \left[\ \boldsymbol{n}_{11}(t),\quad \boldsymbol{n}_{12}(t),\quad \dots\quad \boldsymbol{n}_{N3}(t)\ \right]$$



$$s(\boldsymbol{x}) = \frac{\langle \boldsymbol{x}, \boldsymbol{u} \rangle_F}{||\boldsymbol{u}||_F\, ||\boldsymbol{x}||_F} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

Harris Statistic

Beam/Stack Correlation Statistic

Q: What is the best way to combine single channel correlations?

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Detection Statistic from Zero-Delay Beamforming

$$s\left(\boldsymbol{x}\right) = \frac{1}{N}\sum_{k=1}^{N} \frac{\langle \boldsymbol{x}_k, \boldsymbol{u}_k \rangle}{||\boldsymbol{u}_k||||\boldsymbol{x}_k||} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

Gibbons Statistic

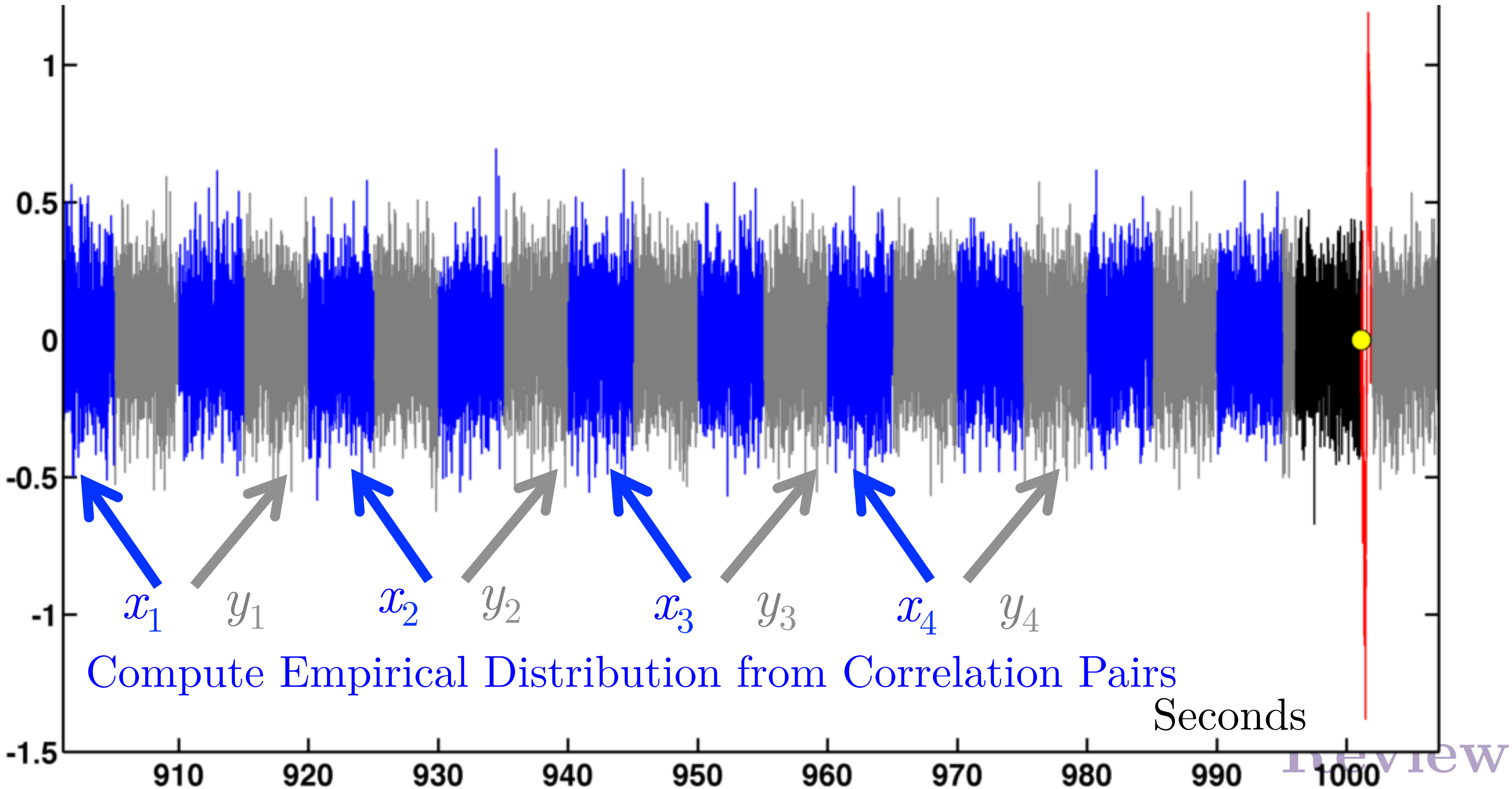
Detection Capability: Does it make a difference what statistic $s(\boldsymbol{x})$ you compute?

Hint: Beaming is better that MLE, if $s(\boldsymbol{x})$ "moderate"...

Estimating Effective N

Q: What is the best way to combine single channel correlations?

Chop up data into non-intersecting windows commensurate with template window length. Select non-neighboring windows at random. Compute $s(\boldsymbol{x})$.



Estimating Effective N

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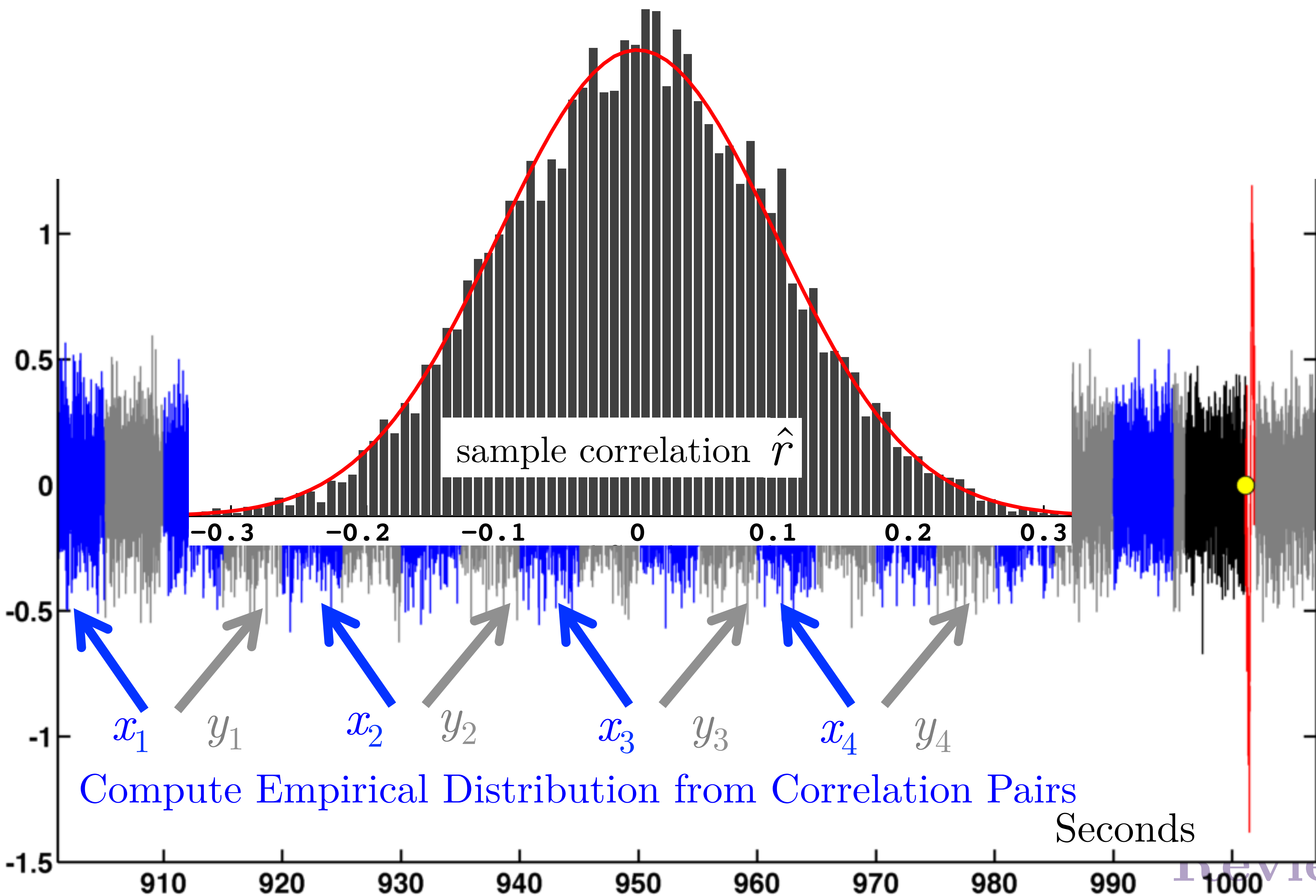


Estimating Effective N

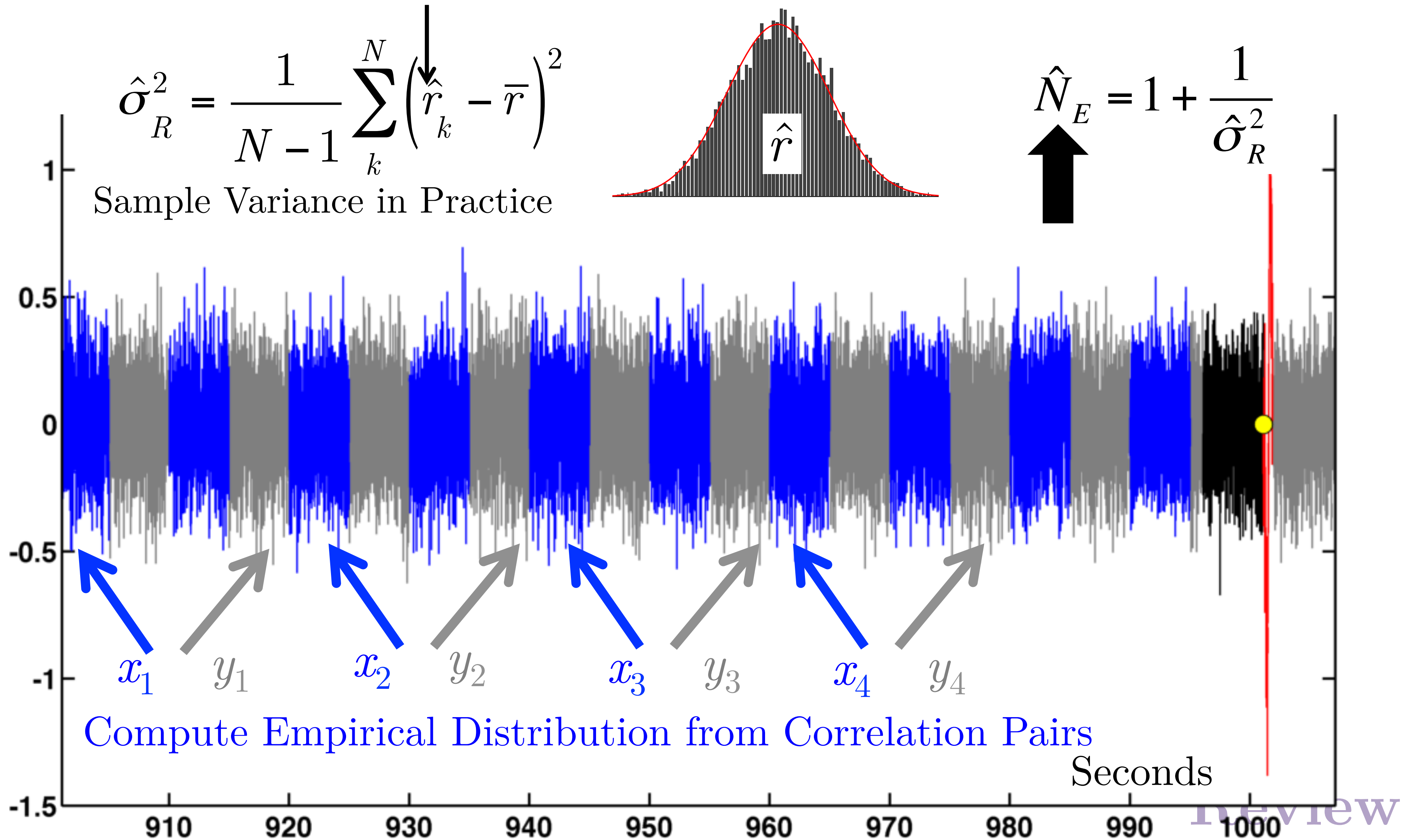
Q: What is the best way to combine single channel correlations?

Empirical sample correlation estimates

$$\hat{\sigma}_R^2 = \frac{1}{N-1}\sum_{k}^{N}\left(\hat{r}_k - \bar{r}\right)^2$$

Sample Variance in Practice

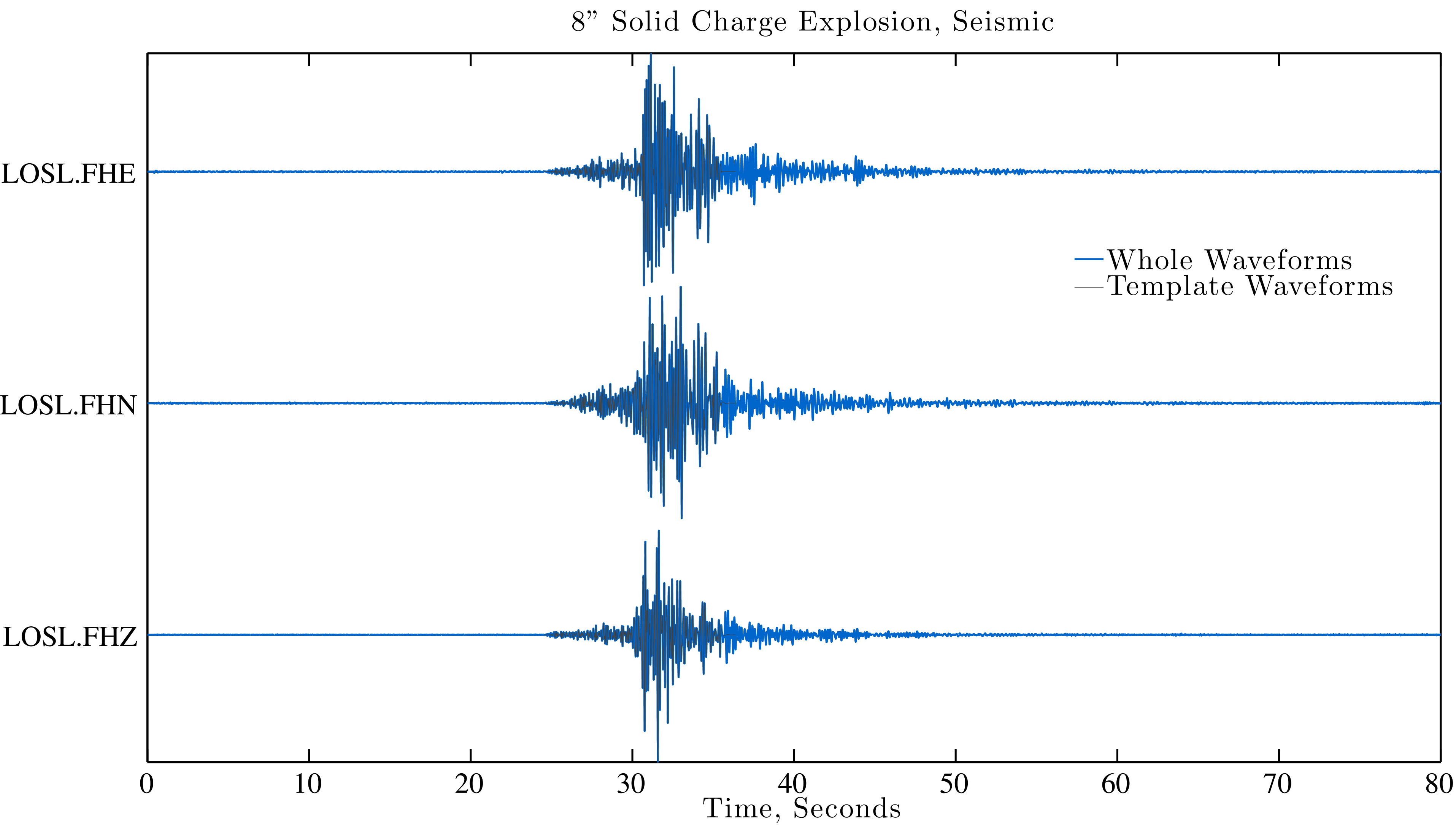
$$\hat{N}_E = 1 + \frac{1}{\hat{\sigma}_R^2}$$



Template Waveform

Q: What is the best way to combine single channel correlations?

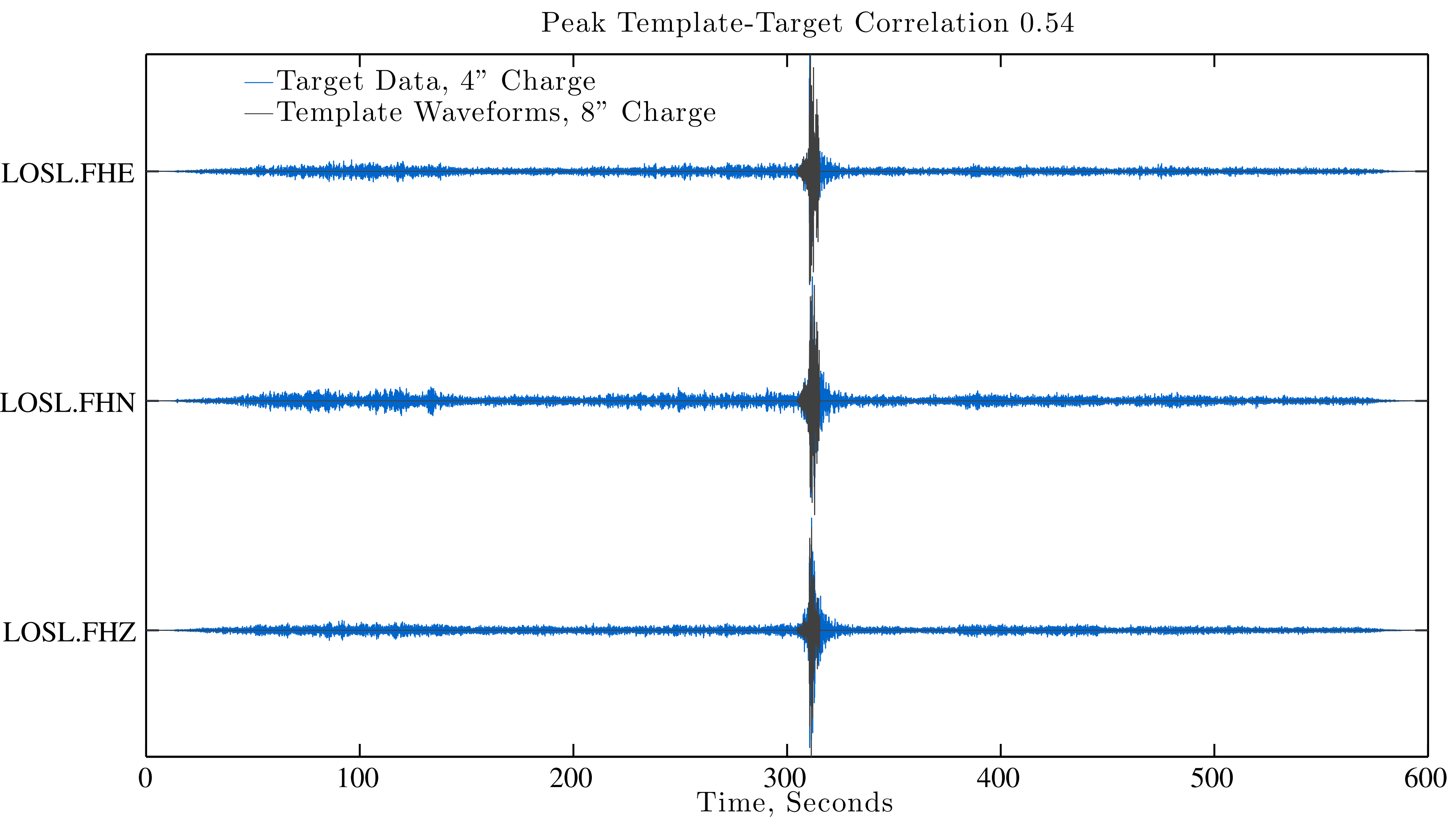
Waveforms recorded during detonation of 8", cylindrical explosive at 1m HOB, local to source



Target Data

Q: What is the best way to combine single channel correlations?

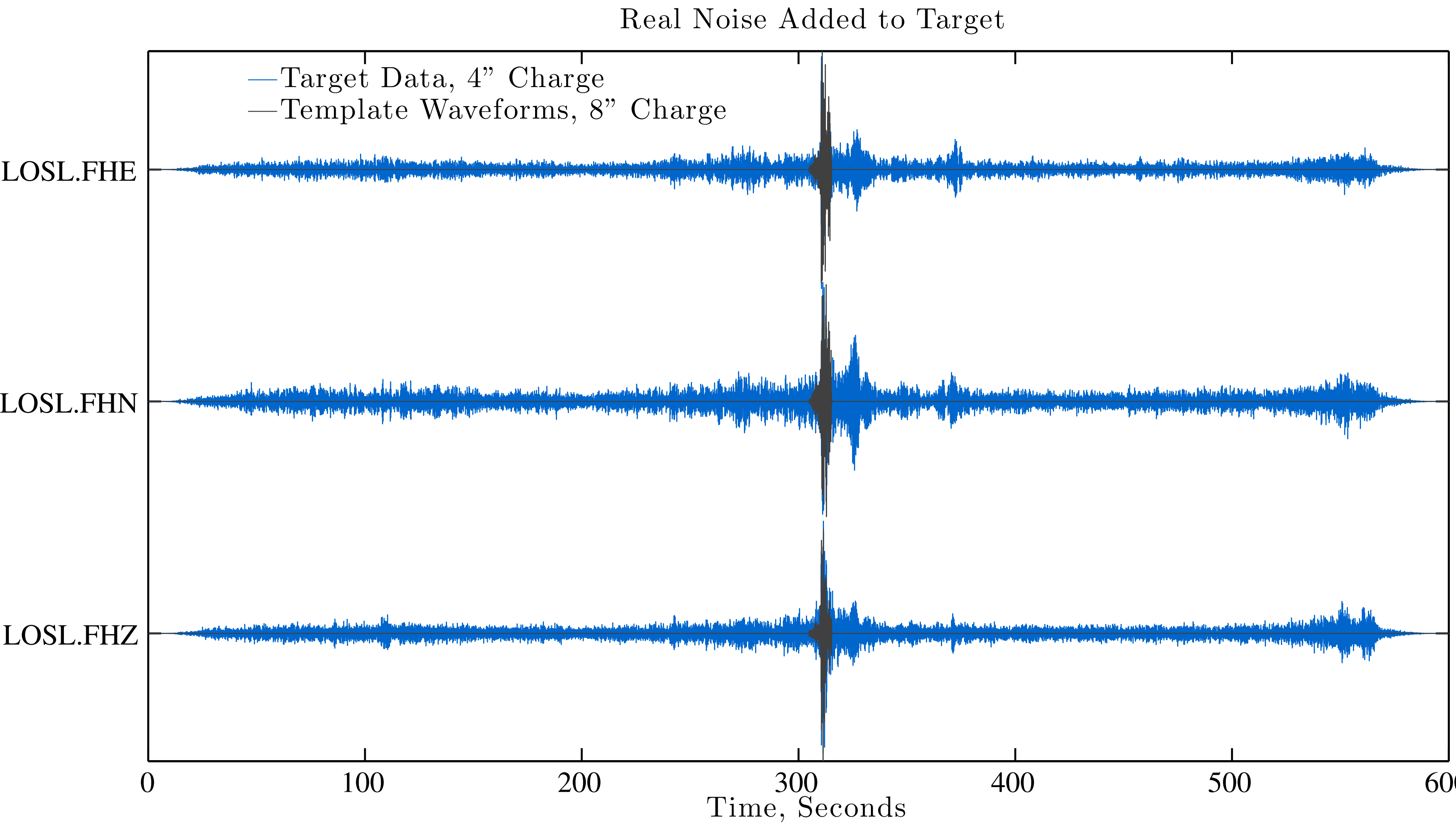
Waveforms recorded during detonation of 4”, cylindrical explosive at 1m HOB, local to source



Target Data

Q: What is the best way to combine single channel correlations?

Waveforms recorded during detonation of 4”, cylindrical explosive at 1m HOB, local to source, **and** add real pre-shot noise to decrease SNR

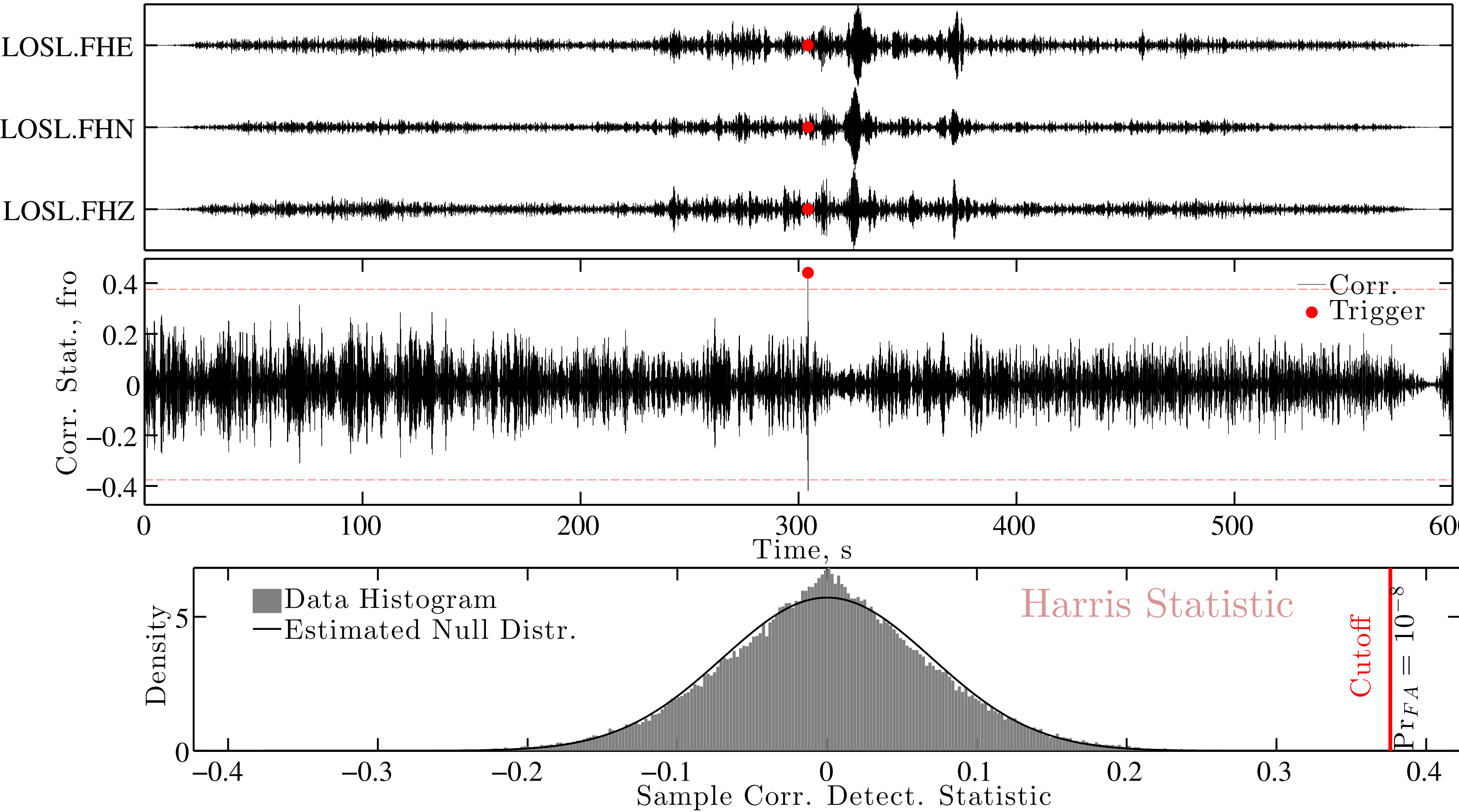


MLE Correlation Statistic

Q: What is the best way to combine single channel correlations?

Scan template over 600 sec of data and use MLE detection statistic

$$s(\boldsymbol{x}) = \frac{\langle \boldsymbol{x}, \boldsymbol{u} \rangle_F}{||\boldsymbol{u}||_F ||\boldsymbol{x}||_F} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

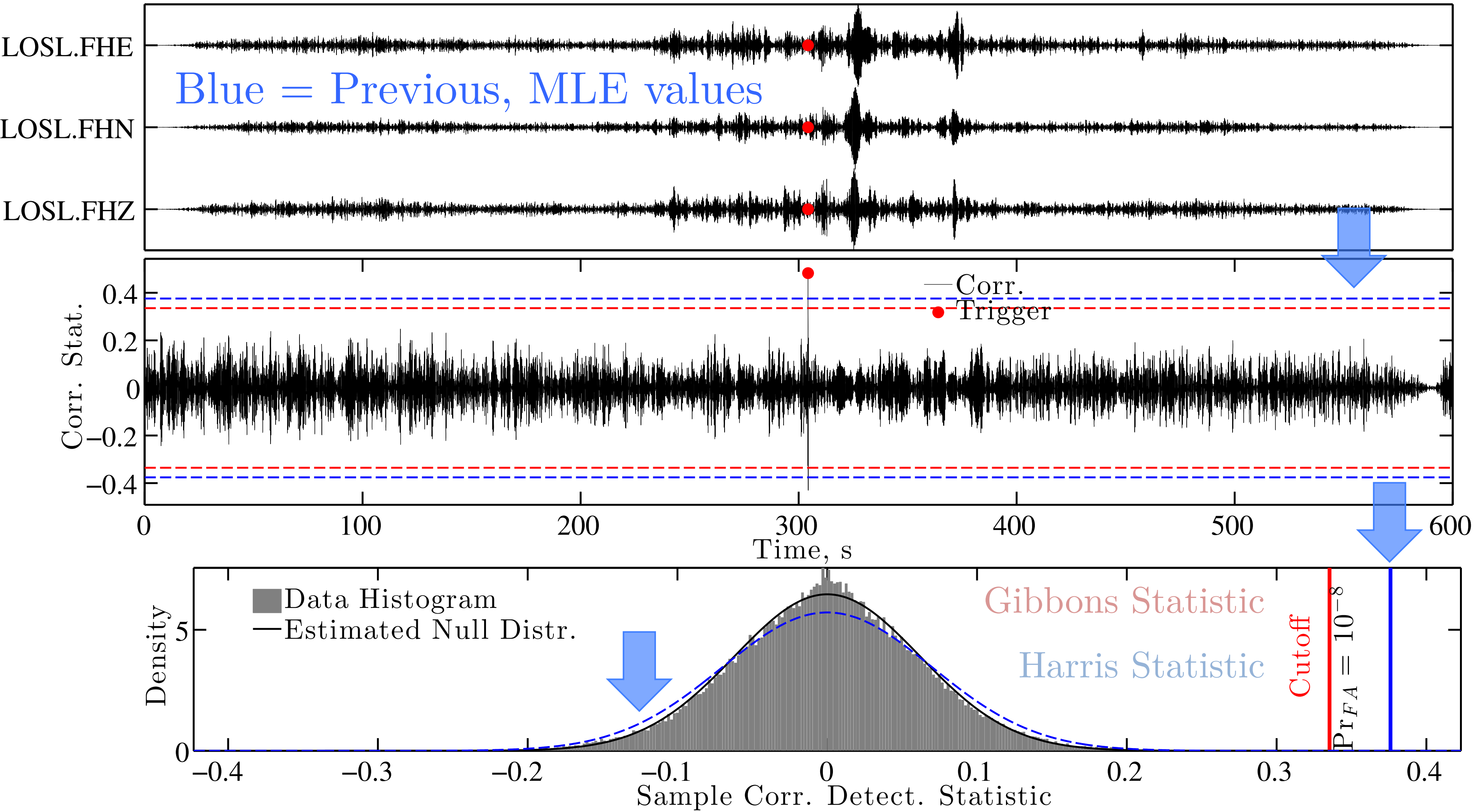


Beam/Stack Correlation Statistic

Q: What is the best way to combine single channel correlations?

Scan template over same data
and average single channel correlation

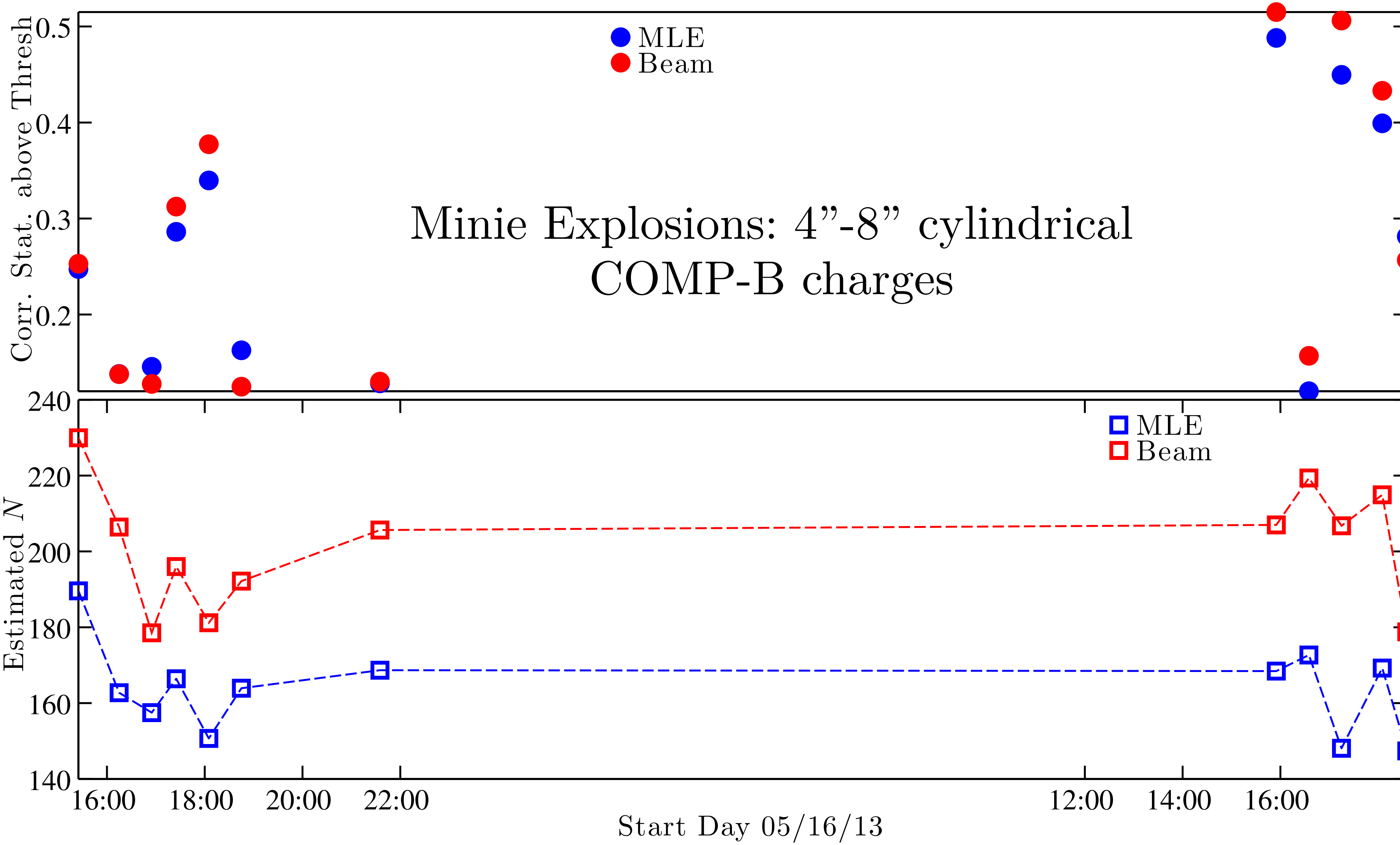
$$s\left(\boldsymbol{x}\right) = \frac{1}{N}\sum_{k=1}^{N}\frac{\langle \boldsymbol{x}_k, \boldsymbol{u}_k \rangle}{||\boldsymbol{u}_k||||\boldsymbol{x}_k||} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$



Compare Detection Thresholds

Q: What is the best way to combine single channel correlations?

Run template over 2 days of data that includes shots:



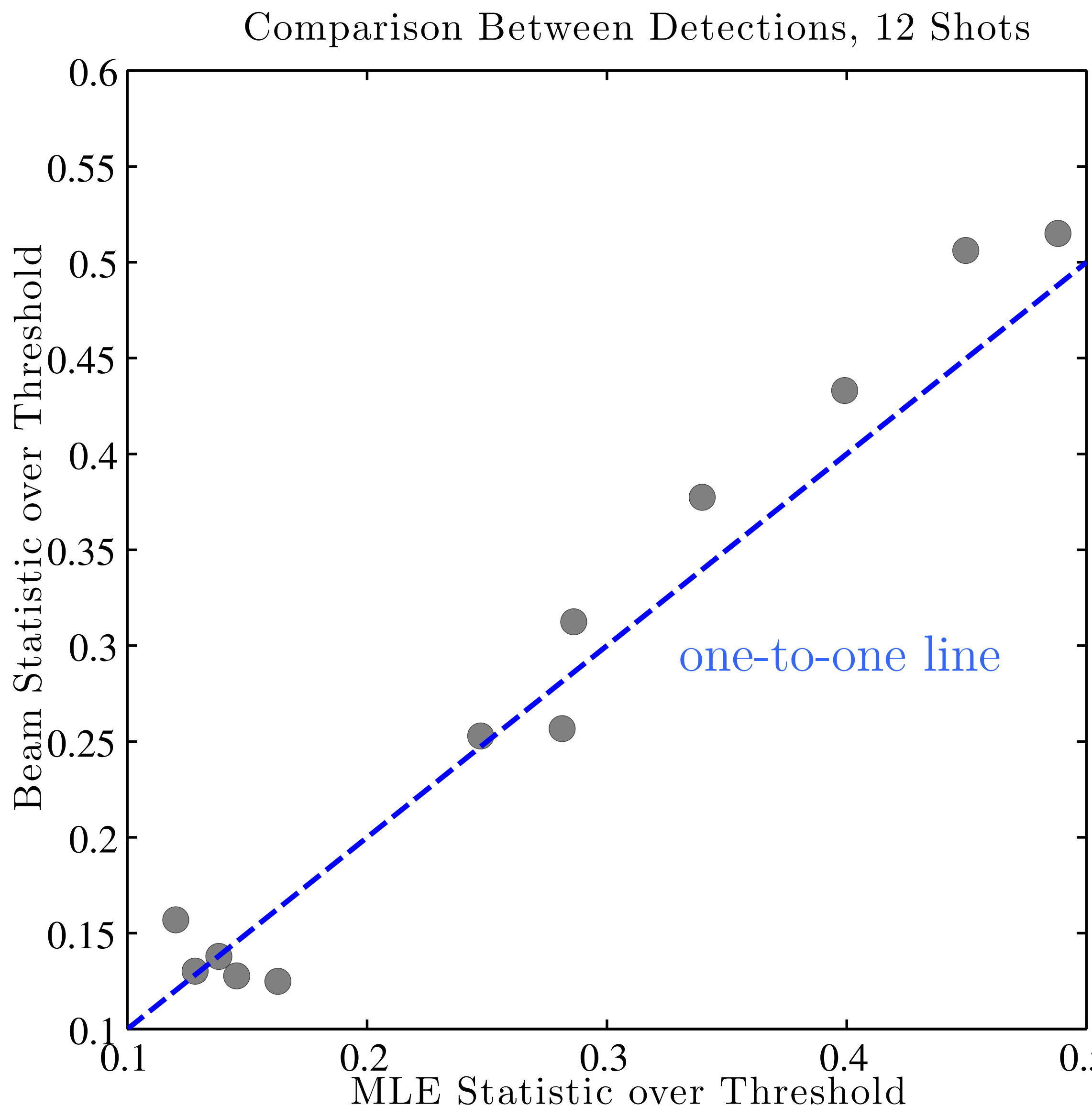
Compare Detection Thresholds

Q: What is the best way to combine single channel correlations?

Detection Values, Relative to False-Alarm Rate Thresholds

Low values of correlation give ambiguous results

At higher values of correlation, summing up single channel statistics gives larger value, *relative to* threshold



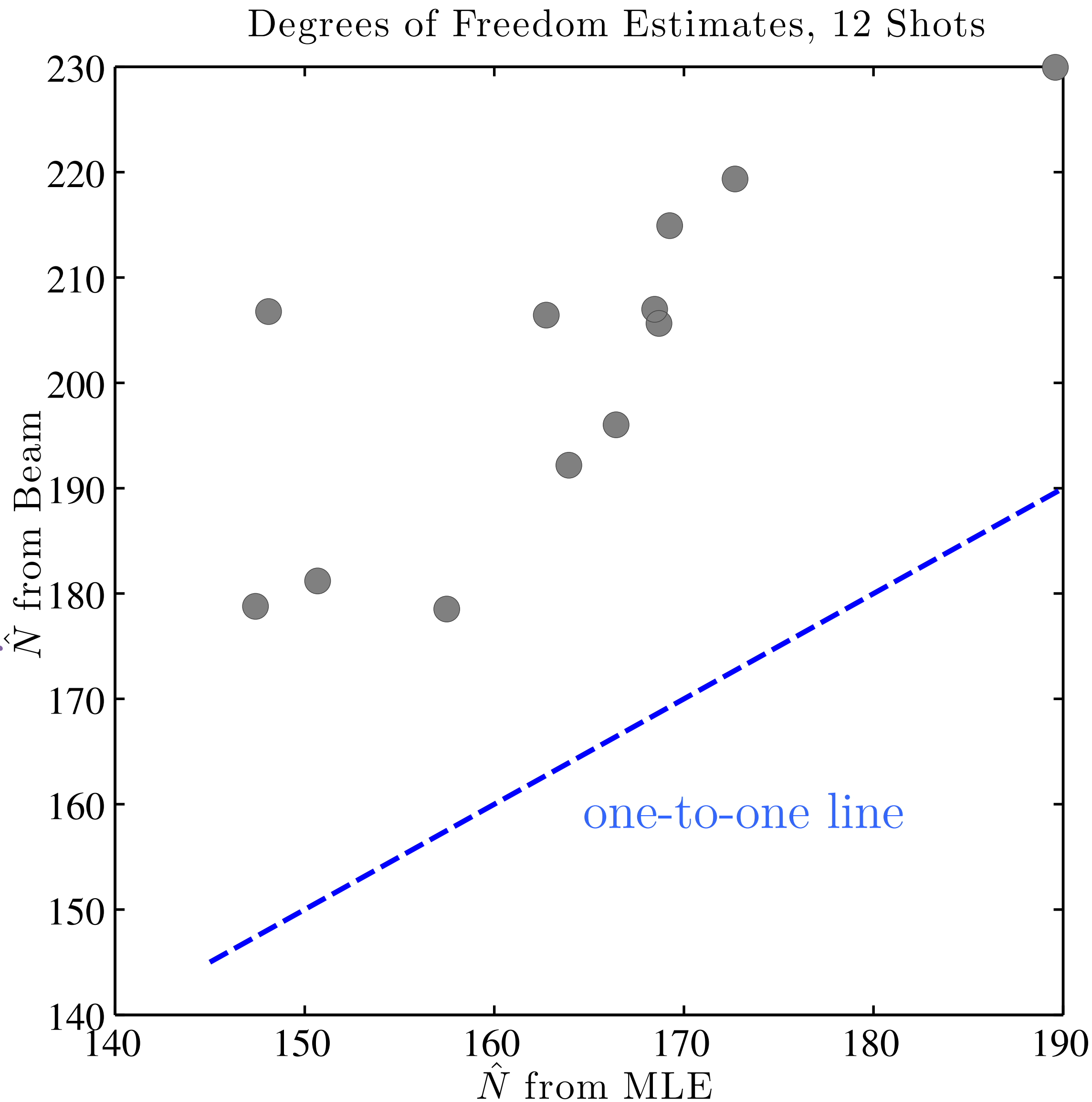
Compare Detection Thresholds

Q: What is the best way to combine single channel correlations?

Effective Degrees of Freedom, Shaping Null Distribution

Null distribution for beamed correlation statistic is always lower variance: distribution is skinner

The Frobenius-norm likely induces statistical dependency between samples in MLE case, and thereby effects denominator



Multichannel Detection Challenges

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1. What is the best way to combine single channel correlations? Can it be demonstrated? ← (Quantitatively, using PDFs)
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Review

Multichannel Detection Challenges

Q: What's the Effect of Uncertainties in template-target on correlation?

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   This is also covered in detection capability talk

3. What if the ambient wavefield isn't composed of noise alone (it's not)?

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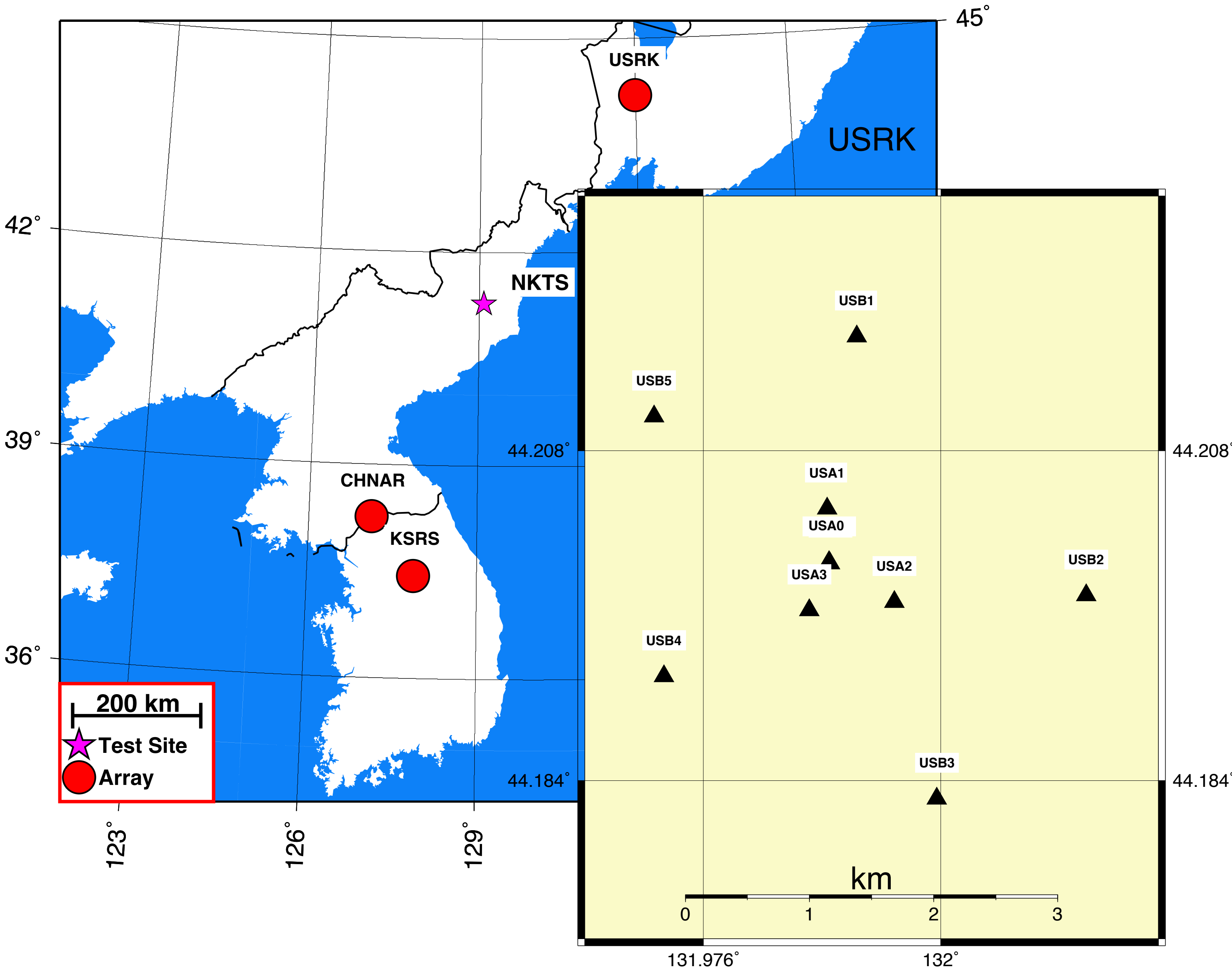
Review

Problem: Detector and Real Data

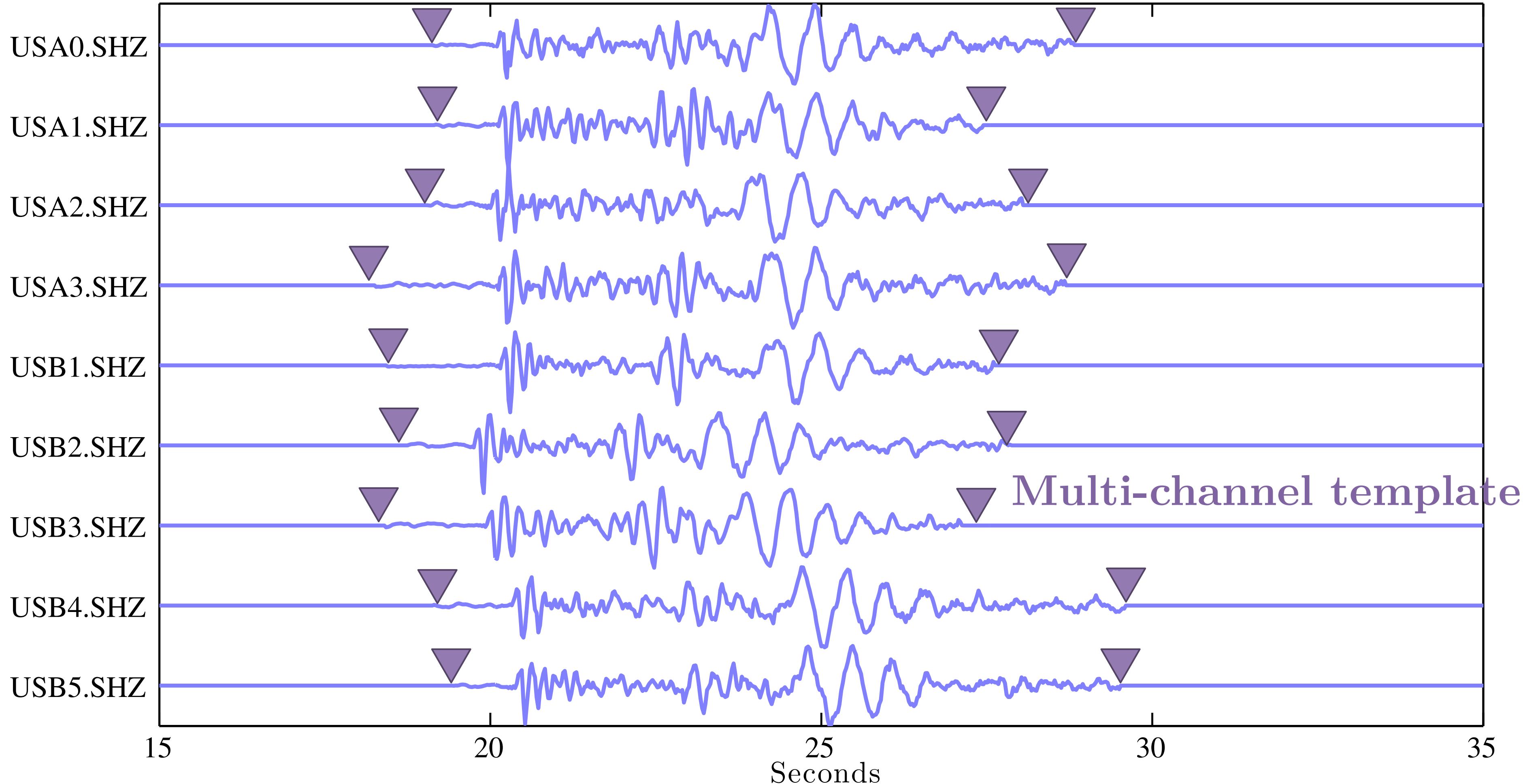
Q: What's the Effect of Uncertainties in template-target on correlation?



Multichannel Template Selection

Q: What's the Effect of Uncertainties in template-target on correlation?

- Run power detector + associate → collect events
- Time-reverse data, re-run power detector, and extract waveform between forward and reverse-picks: ▼
- Cluster using correlation coefficients, select "favorite" signal



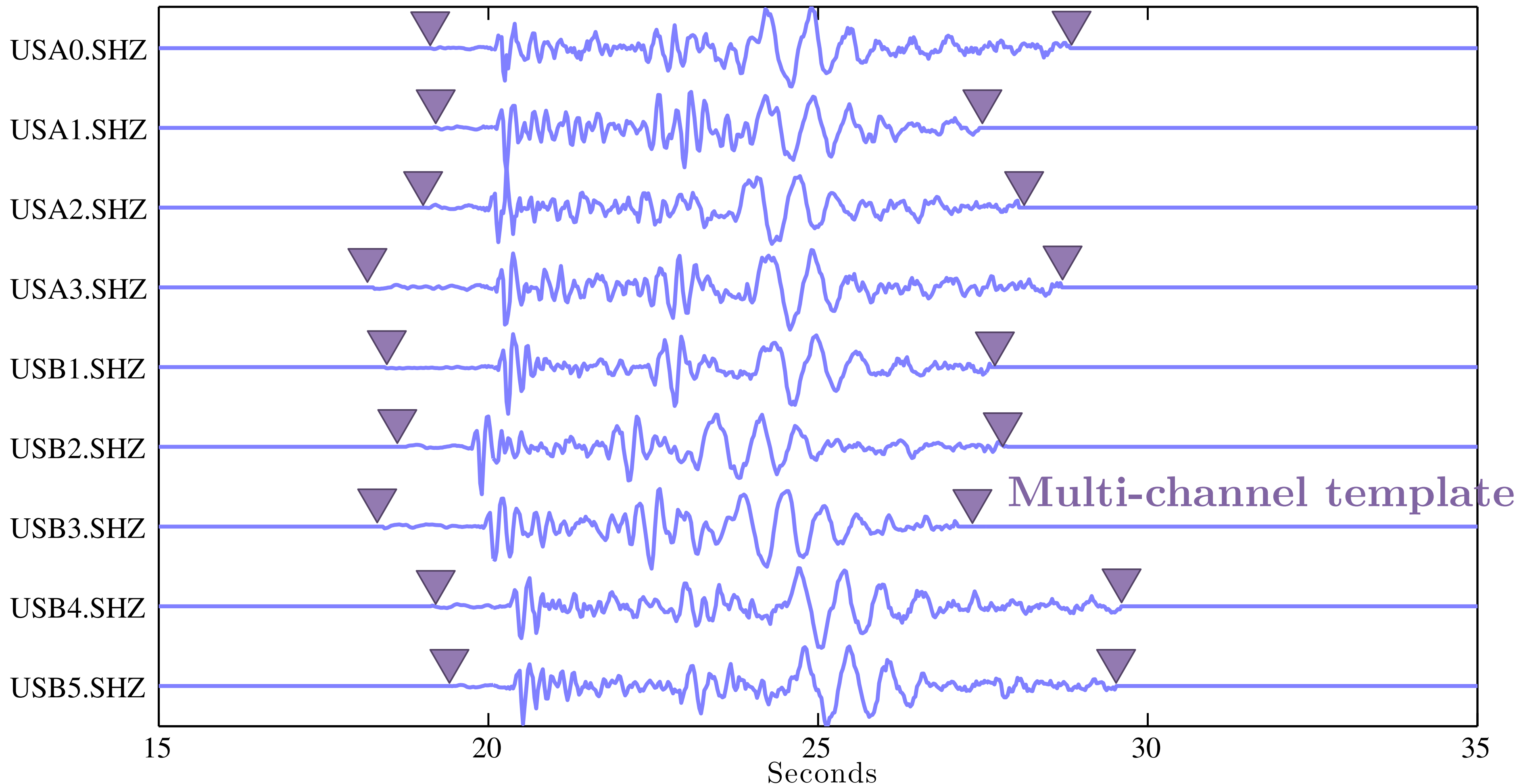
Multichannel Template Selection

Q: What's the Effect of Uncertainties in template-target on correlation?

-
-

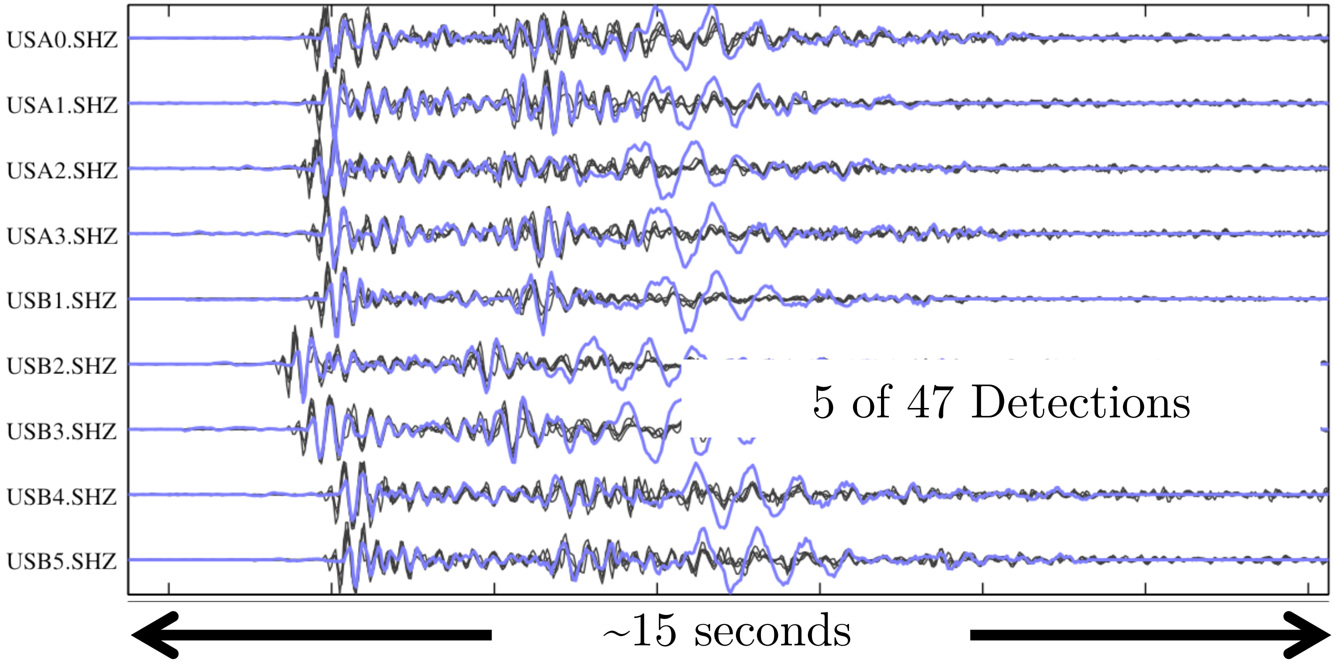
Take these details for granted, and run detector on ≥ 2 months of continuous data

- Cluster using correlation coefficients, select "favorite" signal



Two Months of Detector-Processing

Q: What's the Effect of Uncertainties in template-target on correlation?



...provides a few analyst-approved detections

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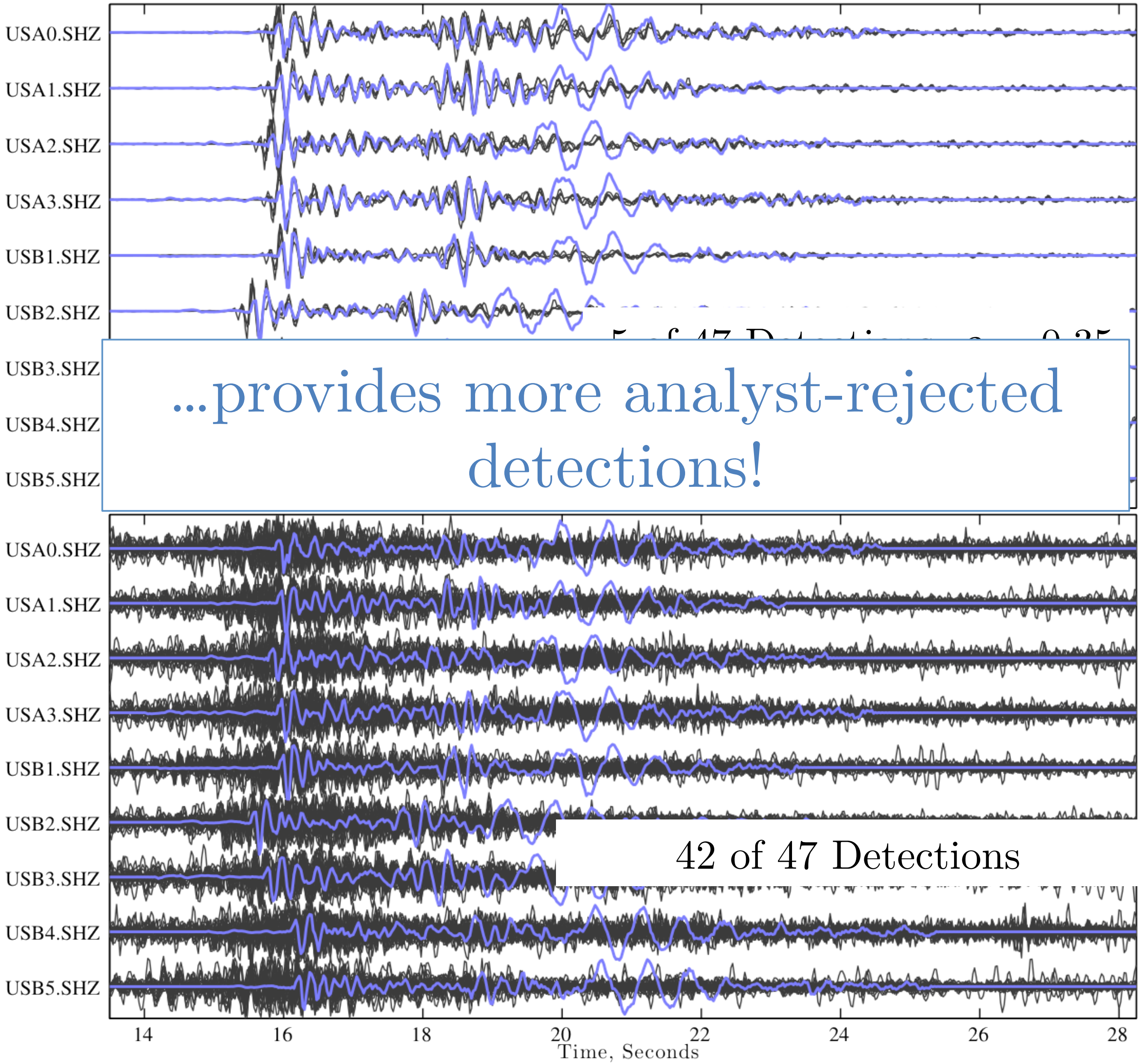

...provides more analyst-rejected detections!

Two Months of Detector-Processing

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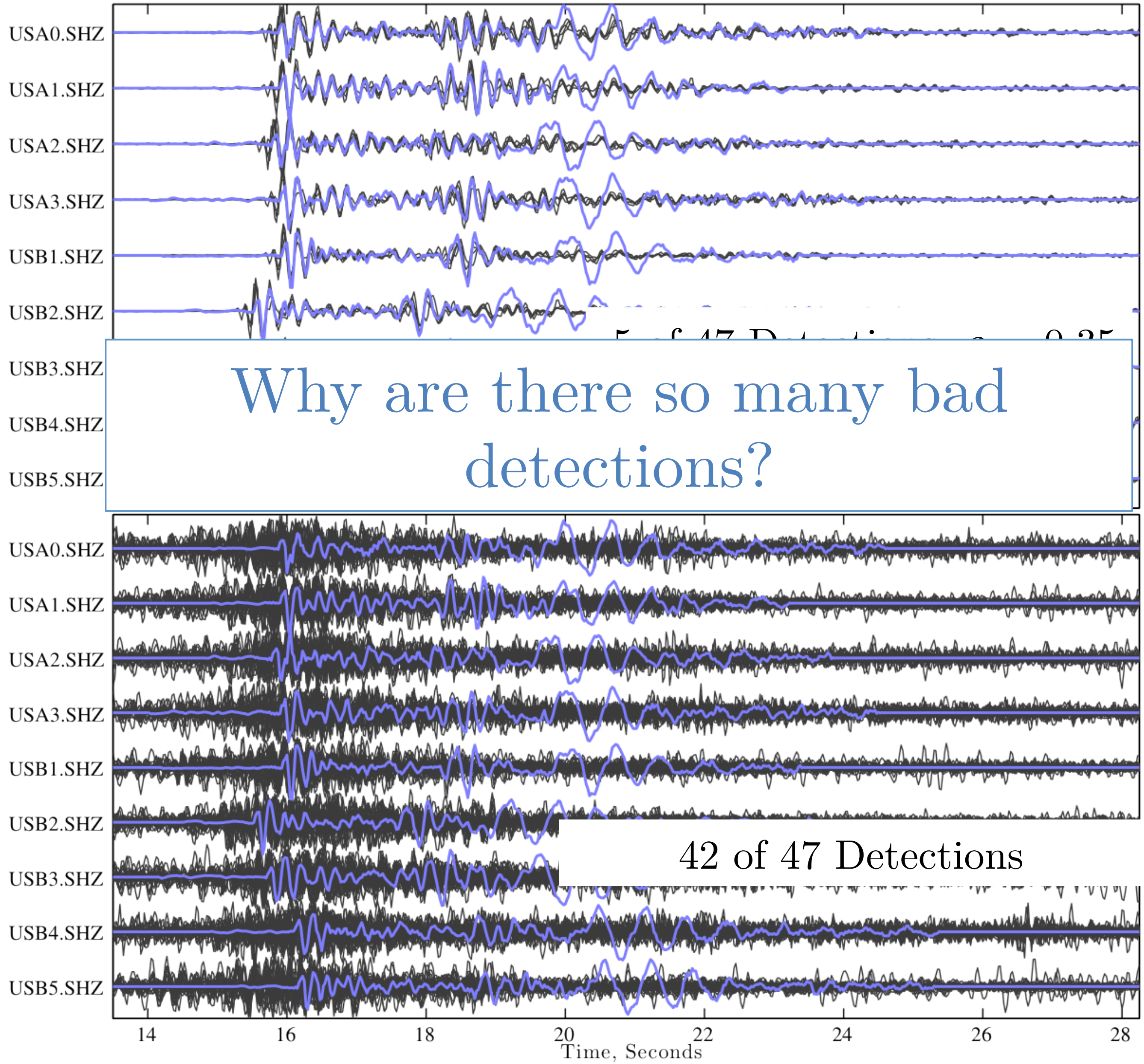

Why are there so many bad detections?

Two Months of Detector-Processing

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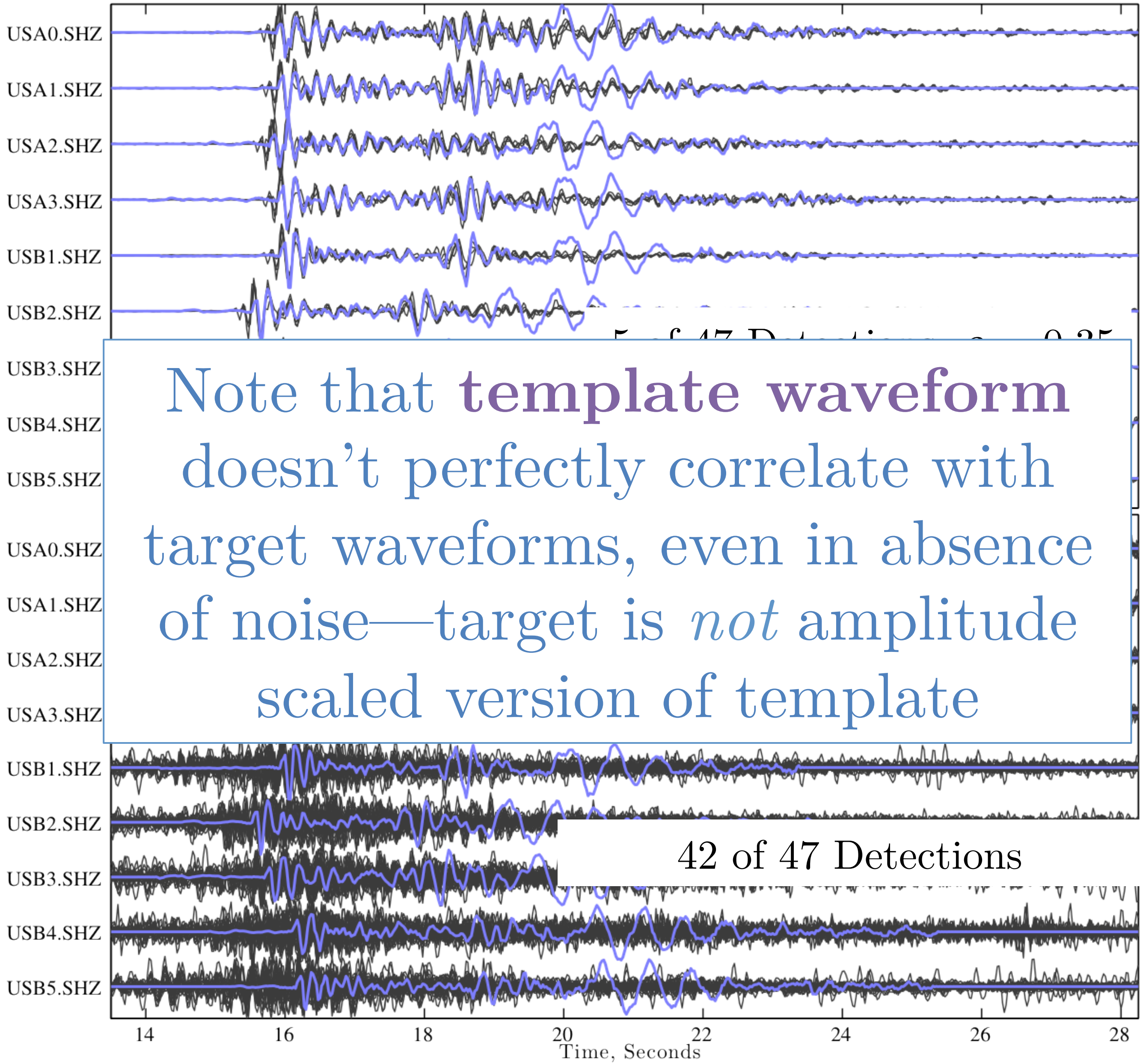


Note that **template waveform** doesn't perfectly correlate with target waveforms, even in absence of noise—target is *not* amplitude scaled version of template

Challenge Summary

Q: What's the Effect of Uncertainties in template-target on correlation?

False detections on non-target waveforms are more frequent than on target waveforms

Non-target detections occur when partially coherent waveforms have suffcient **projection** onto the template signal to exceed the threshold for event declaration

The null hypothesis do not predict presence of dissimilar waveforms

Solutions Requirements

Q: What's the Effect of Uncertainties in template-target on correlation?

False detections are statistically predictable with prescribed false alarm rates

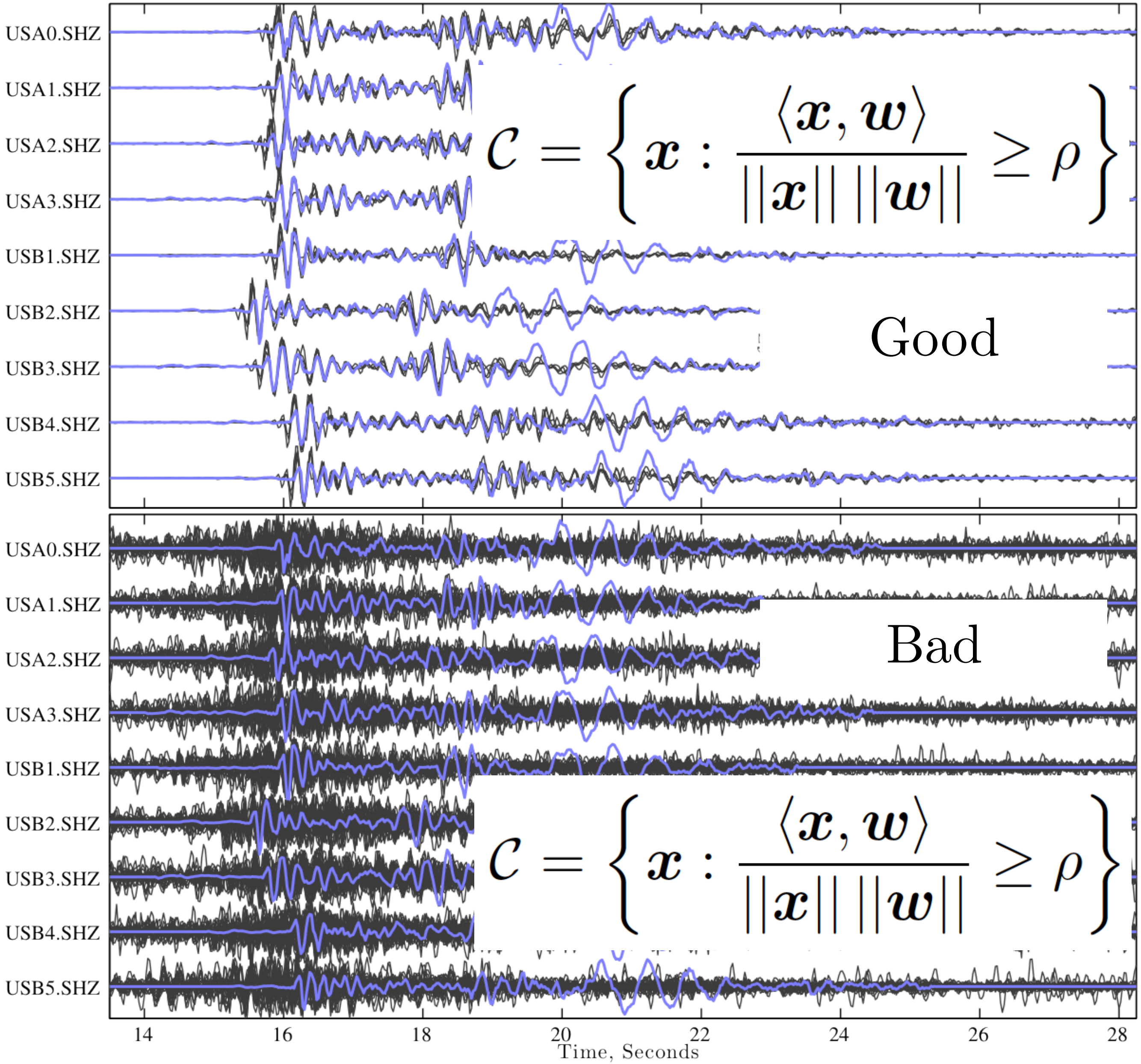
Non-target waveforms should produce a sub-threshold statistic

Revised detector should require minimal modification to current correlation detectors to accomplish objective.

Detection performance must be quantitatively verifiable

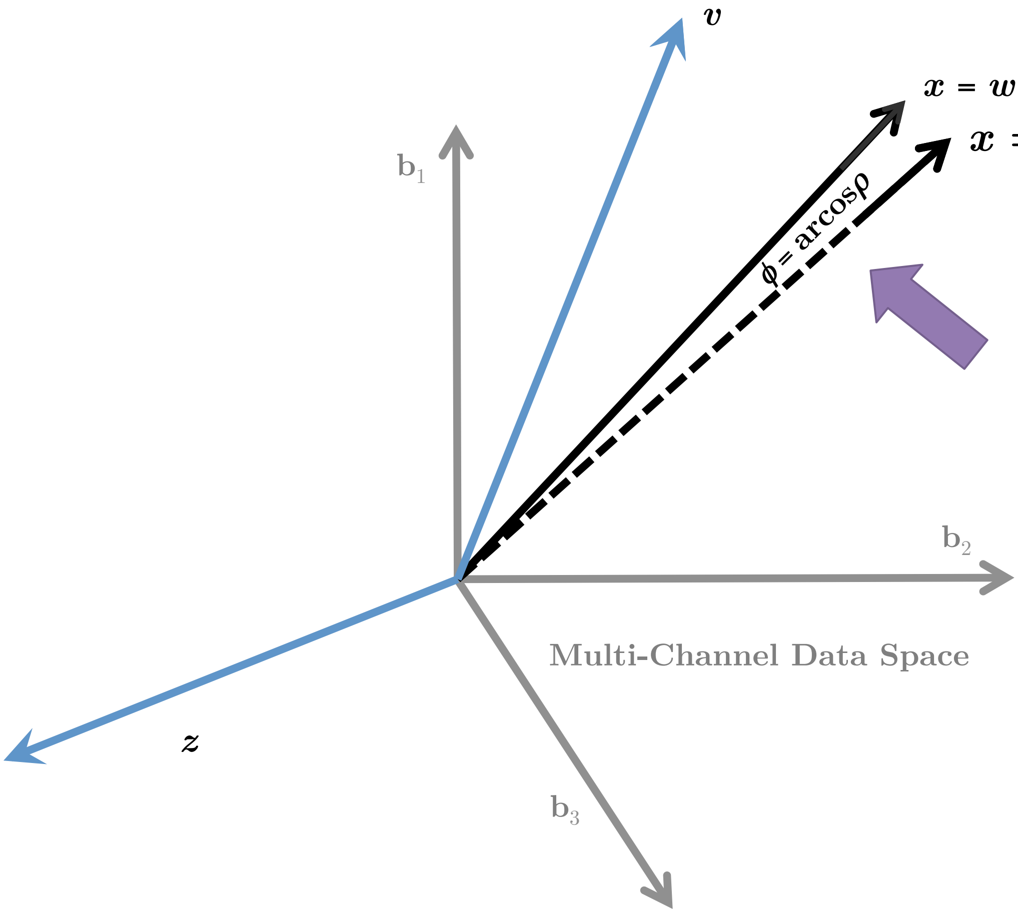
Target and Non-Target Commonality

Q: What's the Effect of Uncertainties in template-target on correlation?



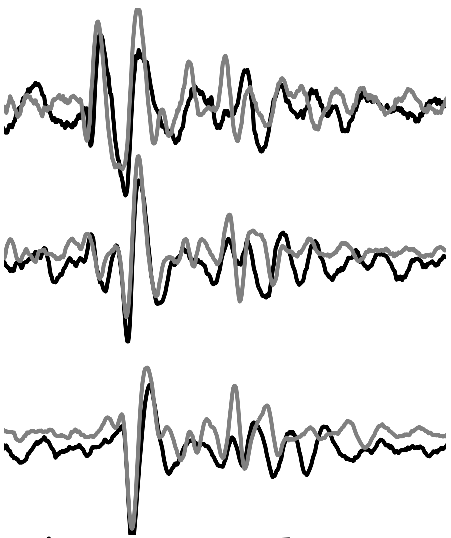
Change Explicit Target Signal Model

Q: What's the Effect of Uncertainties in template-target on correlation?



$x = Aw + n$: data

$$\Pr\{r > \eta\} = \Pr\left\{\frac{x^{T}w}{||x||\,||w||} > \eta\right\}$$



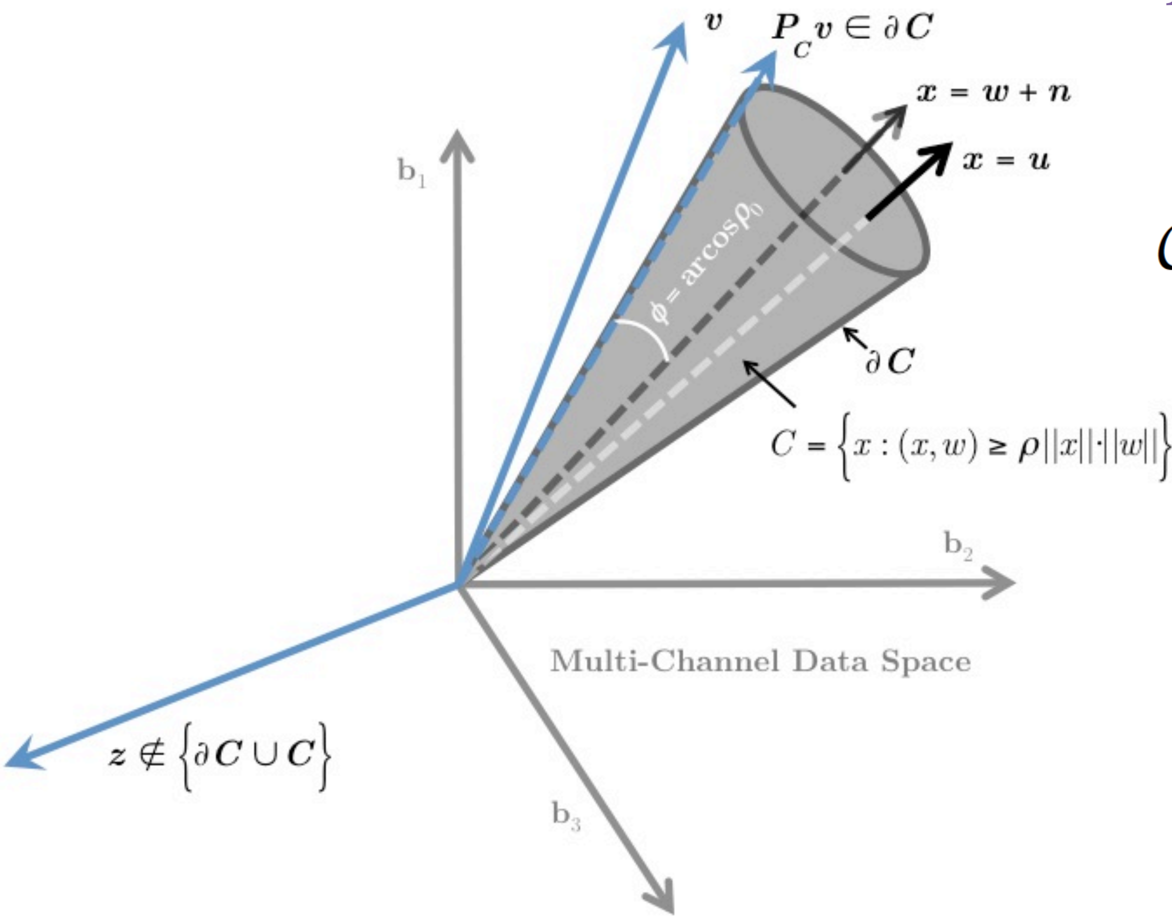
Geometric view of Correlation Detector

$\mathcal{H}_1$: $x = Aw + n$: data stream
$x = w + n$: template

Known Signal in this Case

To an Implicit Signal Target Signal Model

Q: What's the Effect of Uncertainties in template-target on correlation?

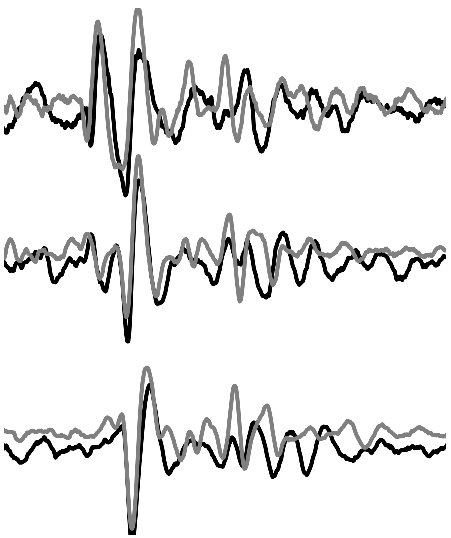


Geometric view of "Cone Detector"

Set contains target and non-target detections with $r \geq \rho$

All detection results contained in C

$$\mathcal{C} = \left\{ x : \frac{\langle x, w \rangle}{||x|| \, ||w||} \geq \rho \right\}$$

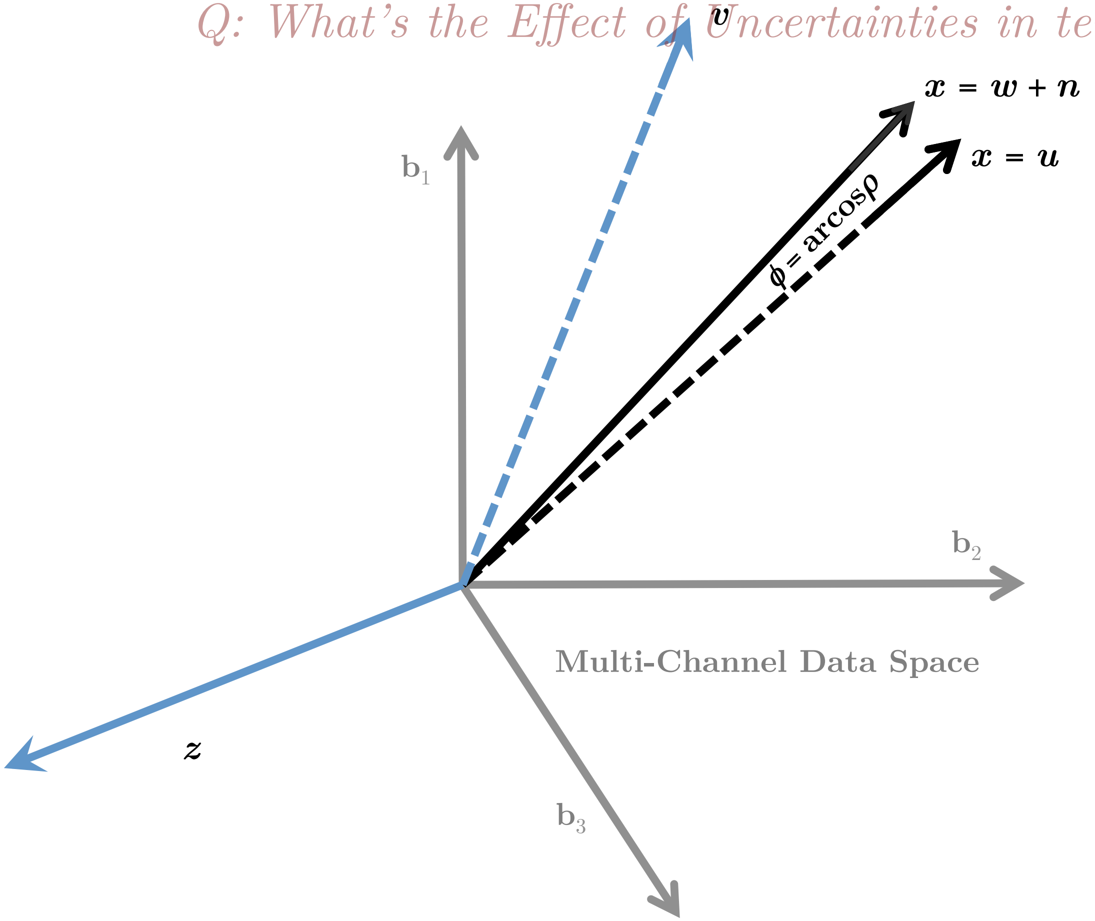


$\mathcal{H}_0$: $x = u + n$: **data stream,** $u \in C$

$x = w + n$: **template**

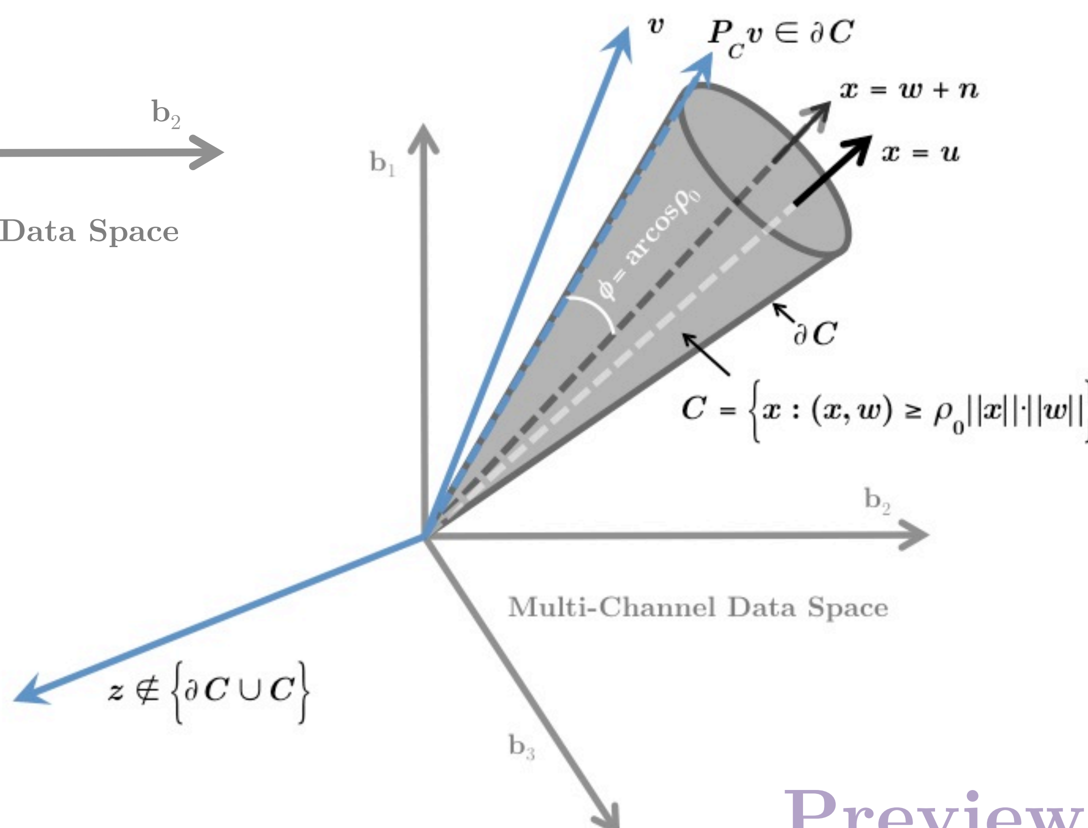
Cone Membership is More Inclusive

Q: What's the Effect of Uncertainties in template-target on correlation?



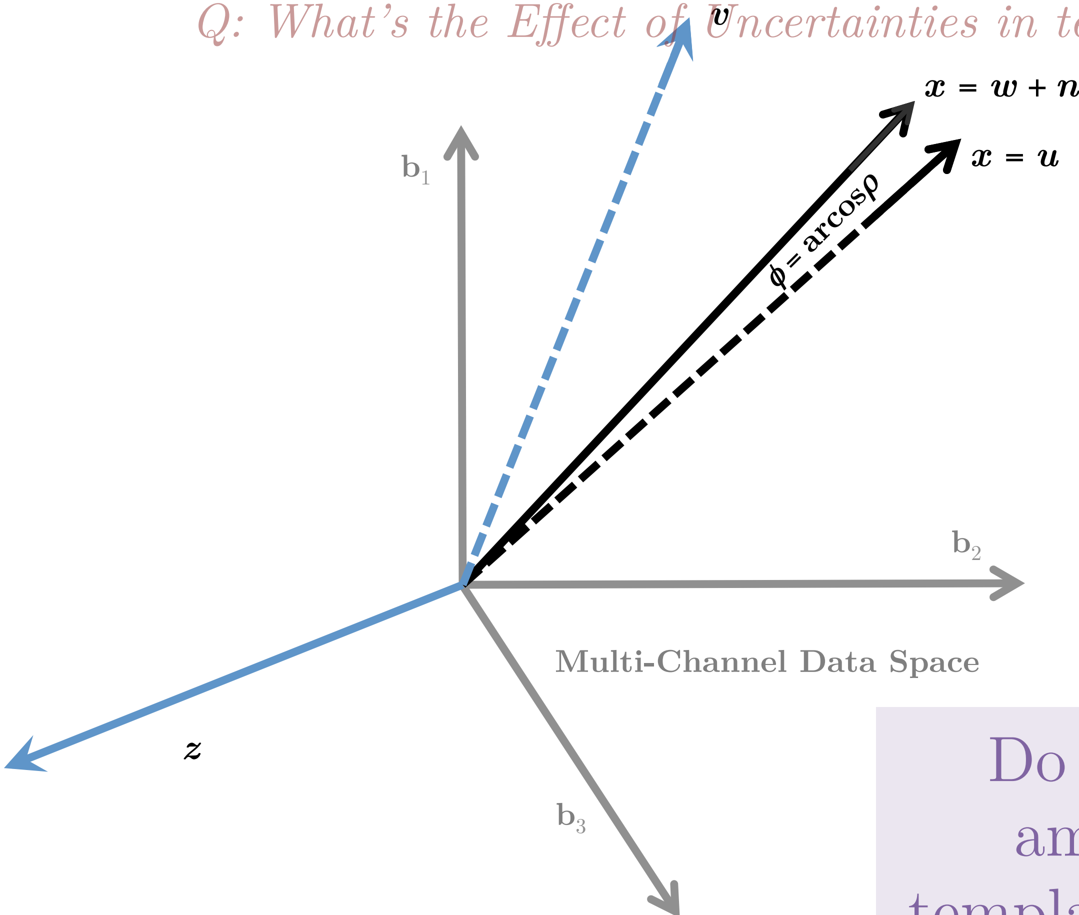
Correlation Detector

Cone Detector



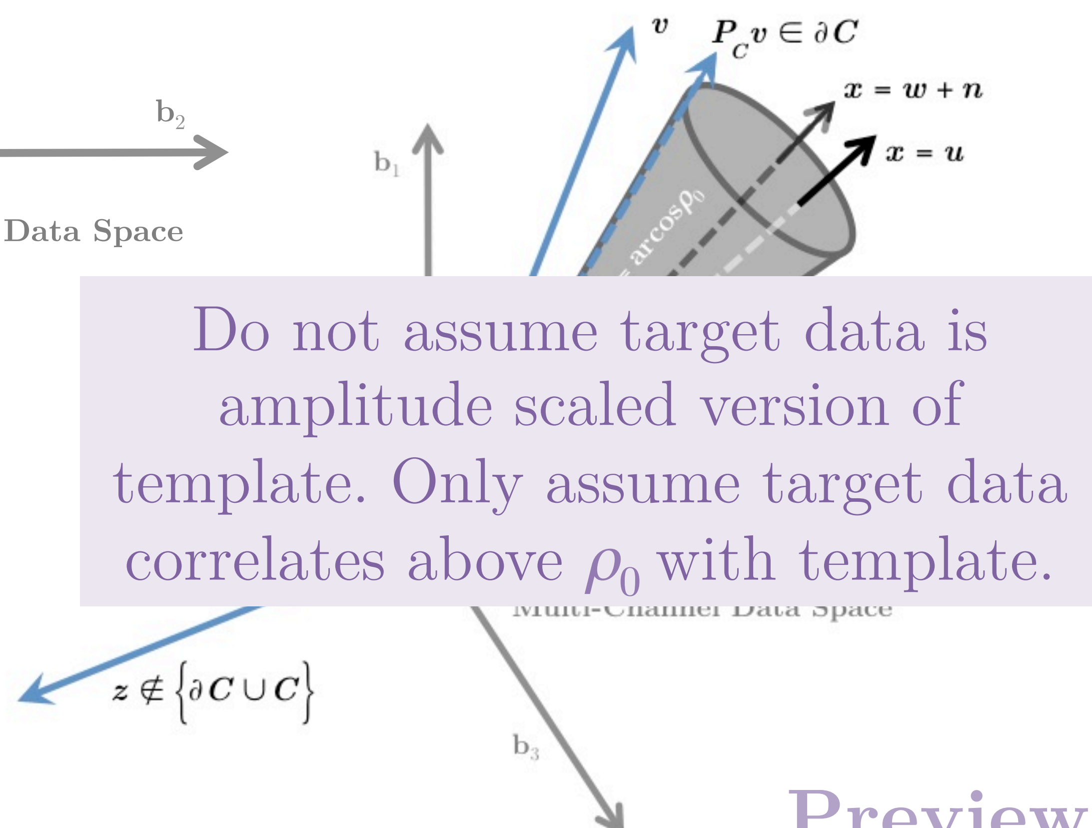
Cone Membership is More Inclusive

Q: What's the Effect of Uncertainties in template-target on correlation?



Correlation Detector

Cone Detector



Do not assume target data is amplitude scaled version of template. Only assume target data correlates above ρ_0 with template.

Preview

Density Function: Cone Detection Statistic

Q: What's the Effect of Uncertainties in template-target on correlation?

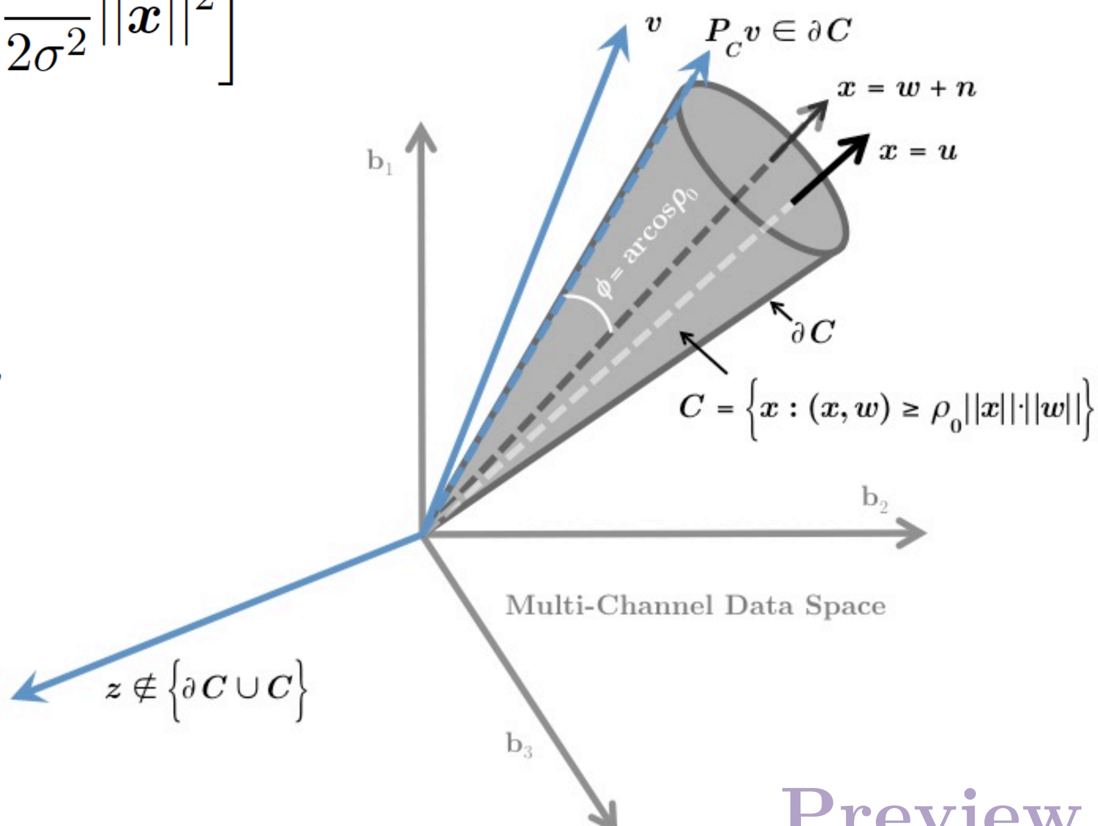
- Form Hypothesis Test with Target Signal in Cone
- Data Still Includes Gaussian Noise

$$p_1(\boldsymbol{x};\mathcal{H}_1) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{2}N_E}} \exp\left[-\frac{1}{2\sigma^2}||\boldsymbol{x}-\boldsymbol{u}||^2\right], \quad \boldsymbol{u} \in \mathcal{C}$$

$$p_0(\boldsymbol{x};\mathcal{H}_1) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{2}N_E}} \exp\left[-\frac{1}{2\sigma^2}||\boldsymbol{x}||^2\right]$$

$$\mathcal{H}_0: \quad \boldsymbol{x} \sim \mathcal{N}\left(\boldsymbol{0}, \sigma^2 \boldsymbol{I}\right)$$

$$\mathcal{H}_1: \quad \boldsymbol{x} \sim \mathcal{N}\left(\boldsymbol{u}, \sigma^2 \boldsymbol{I}\right), \quad \boldsymbol{u} \in \mathcal{C}$$



Preview

Density Function: Cone Detection Statistic

Q: What's the Effect of Uncertainties in template-target on correlation?

- Variances σ^2 and target signal $\boldsymbol{u}$ are imperfectly known
- Form Maximum Likelihood Ratio, $\Lambda(\boldsymbol{x})$

$$p_1(\boldsymbol{x};\, \mathcal{H}_1) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{2}N_E}} \exp\left[-\frac{1}{2\sigma^2}||\boldsymbol{x}-\boldsymbol{u}||^2\right], \quad \boldsymbol{u} \in \mathcal{C}$$

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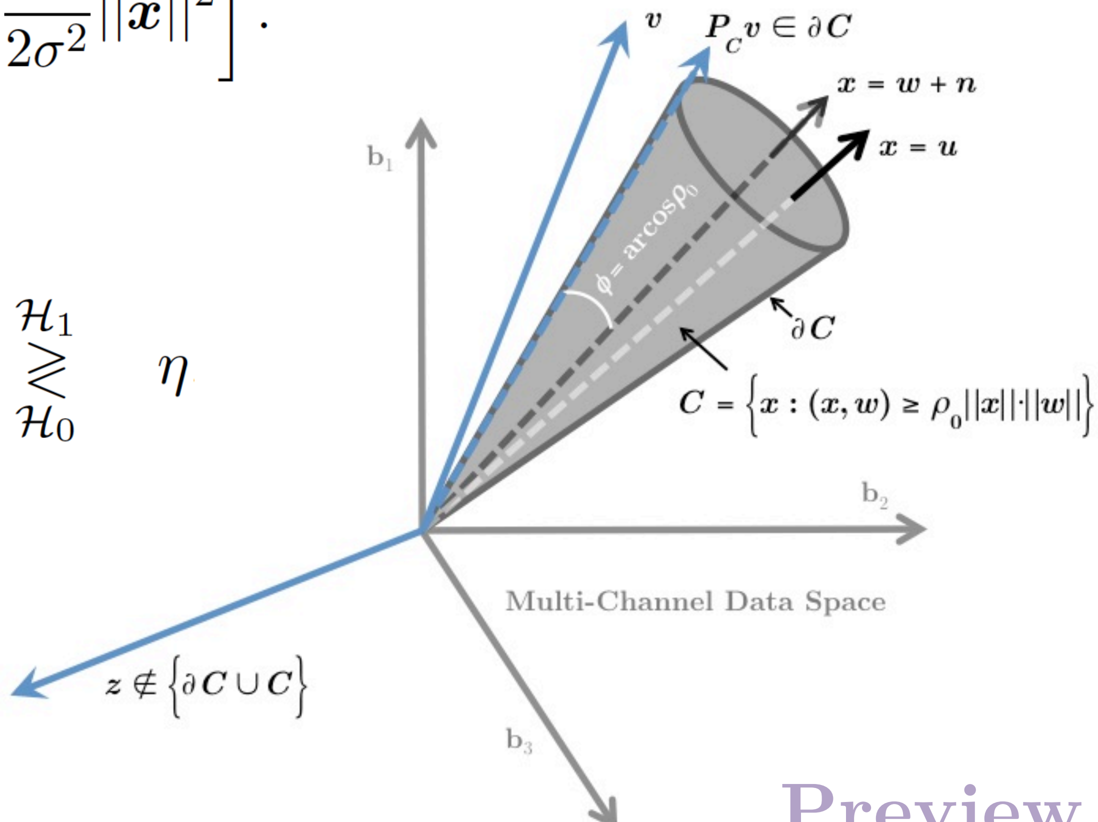
$$\Lambda(\boldsymbol{x}) = \frac{\max\limits_{\sigma,\boldsymbol{u}}\{\, p_1(\boldsymbol{x};\, \mathcal{H}_1)\,\}}{\max\limits_{\sigma}\{\, p_0(\boldsymbol{x};\, \mathcal{H}_0)\,\}} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$



Density Function: Cone Detection Statistic

Q: What's the Effect of Uncertainties in template-target on correlation?

- Substitute maximum likelihood estimators back into $\Lambda(\boldsymbol{x})$
- $\Lambda(\boldsymbol{x})$ reduces to a statistic $s(\boldsymbol{x})$ = projected energy ratio

$$p_1(\boldsymbol{x};\mathcal{H}_1) = \frac{1}{(2\pi\sigma^2)^{\frac{1}{2}N_E}} \exp\left[-\frac{1}{2\sigma^2}||\boldsymbol{x}-\boldsymbol{u}||^2\right], \quad \boldsymbol{u}\in\mathcal{C}$$

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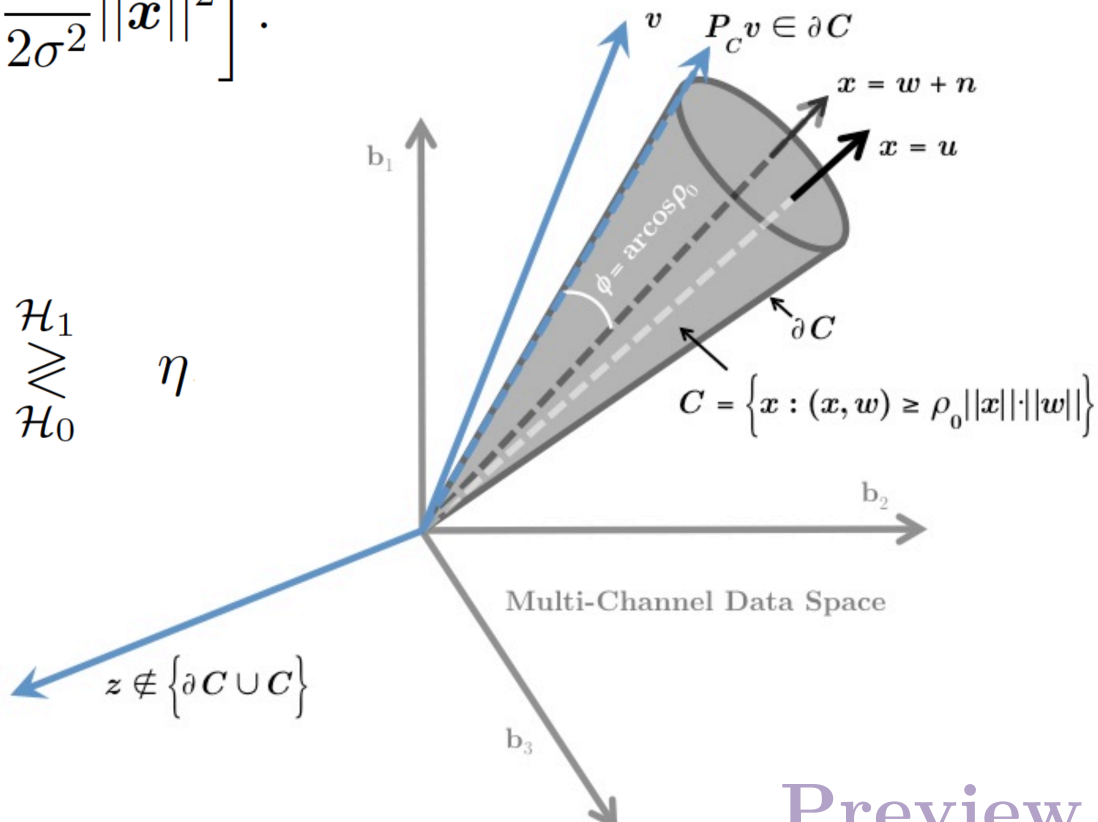

$$\Lambda(\boldsymbol{x}) = \frac{\max_{\sigma,\boldsymbol{u}}\{p_1(\boldsymbol{x};\mathcal{H}_1)\}}{\max_{\sigma}\{p_0(\boldsymbol{x};\mathcal{H}_0)\}} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

$$\frac{||P_{\mathcal{C}}(\boldsymbol{x})||^2}{||\boldsymbol{x}||^2} = s(\boldsymbol{x}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

Preview

Density Function: Cone Detection Statistic

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- Substitute maximum likelihood estimators back into $\Lambda(\boldsymbol{x})$
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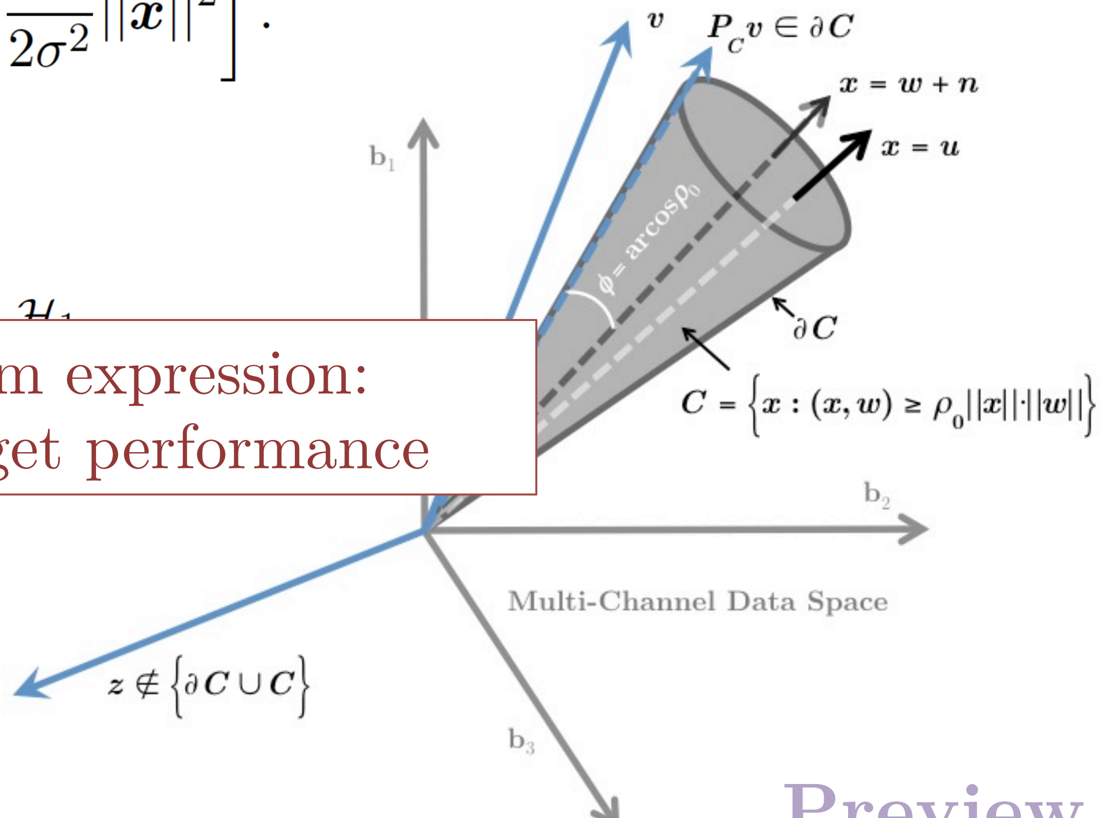
$\max\{\, p_1(\boldsymbol{x};\, \mathcal{H}_1)\,\}$ $\mathcal{H}_1$

PDF for $s(\boldsymbol{x})$ has closed form expression:
no Monte Carlo needed to get performance

$$\frac{||P_{\mathcal{C}}(\boldsymbol{x})||^2}{||\boldsymbol{x}||^2} = s(\boldsymbol{x}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$



Preview

Density Function: Cone Detection Statistic

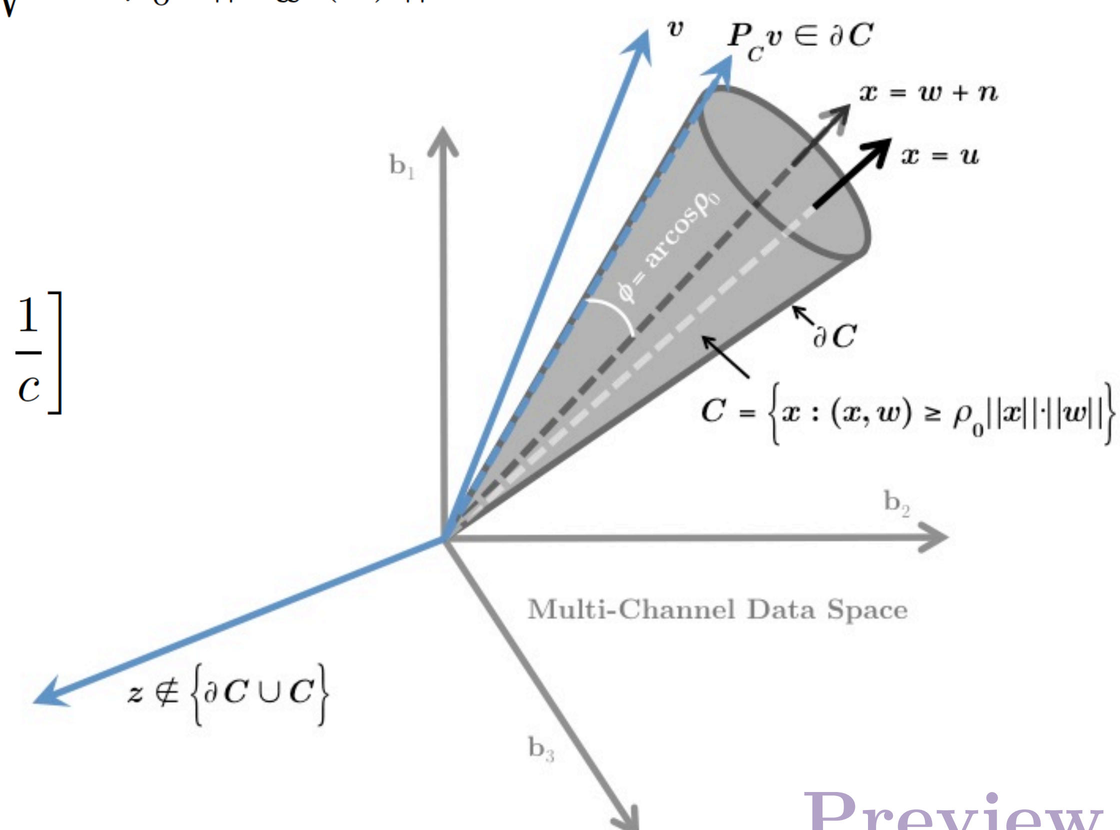
Q: What's the Effect of Uncertainties in template-target on correlation?

- Statistic $s(\boldsymbol{x})$ is nonlinear, and conditional upon correlation r
- Statistic "compresses" decision region into [-c, ρ_0]
- Statistic is function of (already computed) sample correlation

$$\gamma \triangleq \rho_0 \langle \boldsymbol{w}, P_{\boldsymbol{w}}(\boldsymbol{x}) \rangle + \sqrt{1-\rho_0^2} \cdot ||P_{\boldsymbol{w}}^{\perp}(\boldsymbol{x})||$$

$$s(\boldsymbol{x}) = \begin{cases} 0 & \frac{r}{\sqrt{1-r^2}} \leq -c \\ \frac{\gamma}{||\boldsymbol{x}||} & \frac{r}{\sqrt{1-r^2}} \in \left[-c, \frac{1}{c}\right] \\ 1 & \frac{r}{\sqrt{1-r^2}} > \frac{1}{c} \end{cases}$$

$$\frac{||P_{\mathcal{C}}(\boldsymbol{x})||^2}{||\boldsymbol{x}||^2} = s(\boldsymbol{x}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$



Density Function: Cone Detection Statistic

Q: What's the Effect of Uncertainties in template-target on correlation?

- Total probability is computed from projection probabilities
- Ratio in r and statistic $s(\boldsymbol{x})$ have determinable PDFs

$$\begin{aligned}\Pr\{s(\boldsymbol{x})>\eta\} = & \Pr\{s(\boldsymbol{x})>\eta \mid P_{\mathcal{C}}(\boldsymbol{x})\in\mathcal{C}\}\cdot\Pr\{P_{\mathcal{C}}(\boldsymbol{x})\in\mathcal{C}\}+\cdots\\ & \Pr\{s(\boldsymbol{x})>\eta \mid P_{\mathcal{C}}(\boldsymbol{x})\in\partial\mathcal{C}\}\cdot\Pr\{P_{\mathcal{C}}(\boldsymbol{x})\in\partial\mathcal{C}\}+\cdots\\ & \Pr\{s(\boldsymbol{x})>\eta \mid P_{\mathcal{C}}(\boldsymbol{x})\notin\mathcal{C}\cup\partial\mathcal{C}\}\cdot\Pr\{P_{\mathcal{C}}(\boldsymbol{x})\notin\mathcal{C}\cup\partial\mathcal{C}\}\end{aligned}$$

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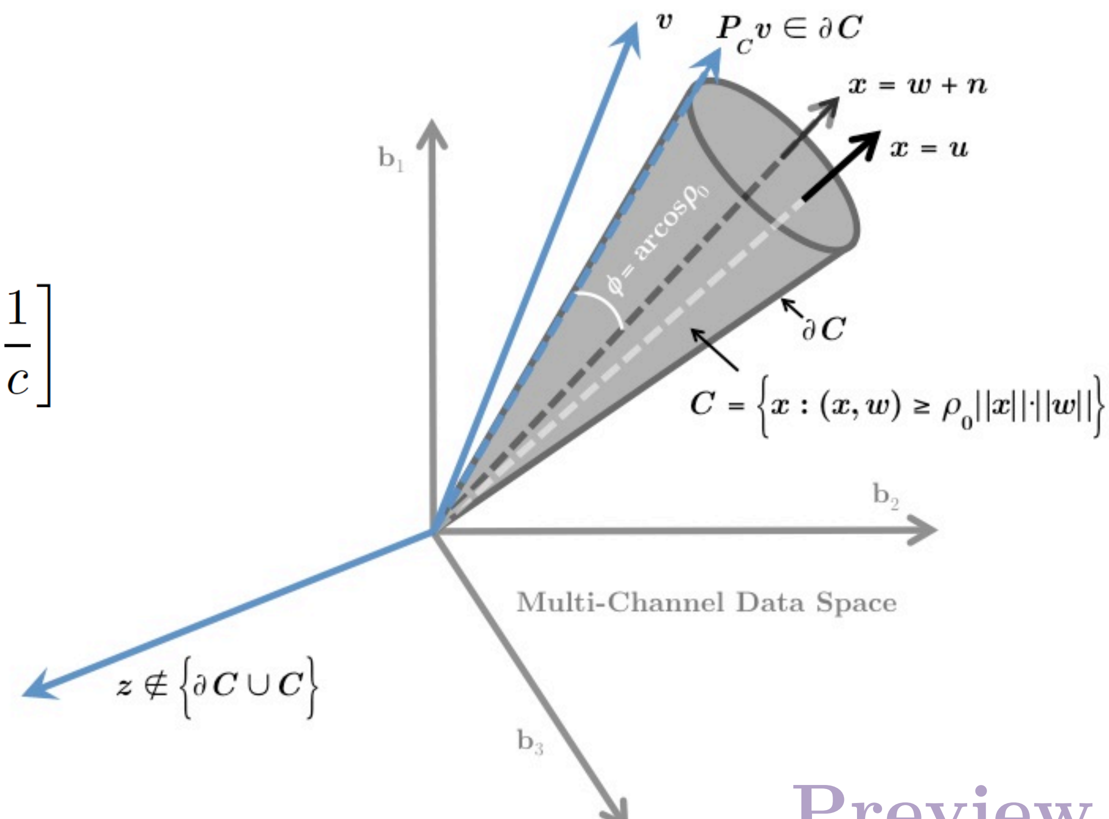
$$\frac{\|P_{\mathcal{C}}(\boldsymbol{x})\|^2}{\|\boldsymbol{x}\|^2} = s(\boldsymbol{x}) \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$



Density Function: Cone Detection Statistic

Q: What's the Effect of Uncertainties in template-target on correlation?

- Variances σ^2 and target signal $\boldsymbol{u}$ are imperfectly known
- Form Maximum Likelihood Ratio, $\Lambda(\boldsymbol{x})$

$$\begin{aligned}\Pr\{s(\boldsymbol{x}) > \eta\} = {} & \Pr\{s(\boldsymbol{x}) > \eta \,|\, P_{\mathcal{C}}(\boldsymbol{x}) \in \mathcal{C}\} \cdot \Pr\{P_{\mathcal{C}}(\boldsymbol{x}) \in \mathcal{C}\} + \cdots \\ & \Pr\{s(\boldsymbol{x}) > \eta \,|\, P_{\mathcal{C}}(\boldsymbol{x}) \in \partial\mathcal{C}\} \cdot \Pr\{P_{\mathcal{C}}(\boldsymbol{x}) \in \partial\mathcal{C}\} + \cdots \\ & \Pr\{s(\boldsymbol{x}) > \eta \,|\, P_{\mathcal{C}}(\boldsymbol{x}) \notin \mathcal{C} \cup \partial\mathcal{C}\} \cdot \Pr\{P_{\mathcal{C}}(\boldsymbol{x}) \notin \mathcal{C} \cup \partial\mathcal{C}\}\end{aligned}$$

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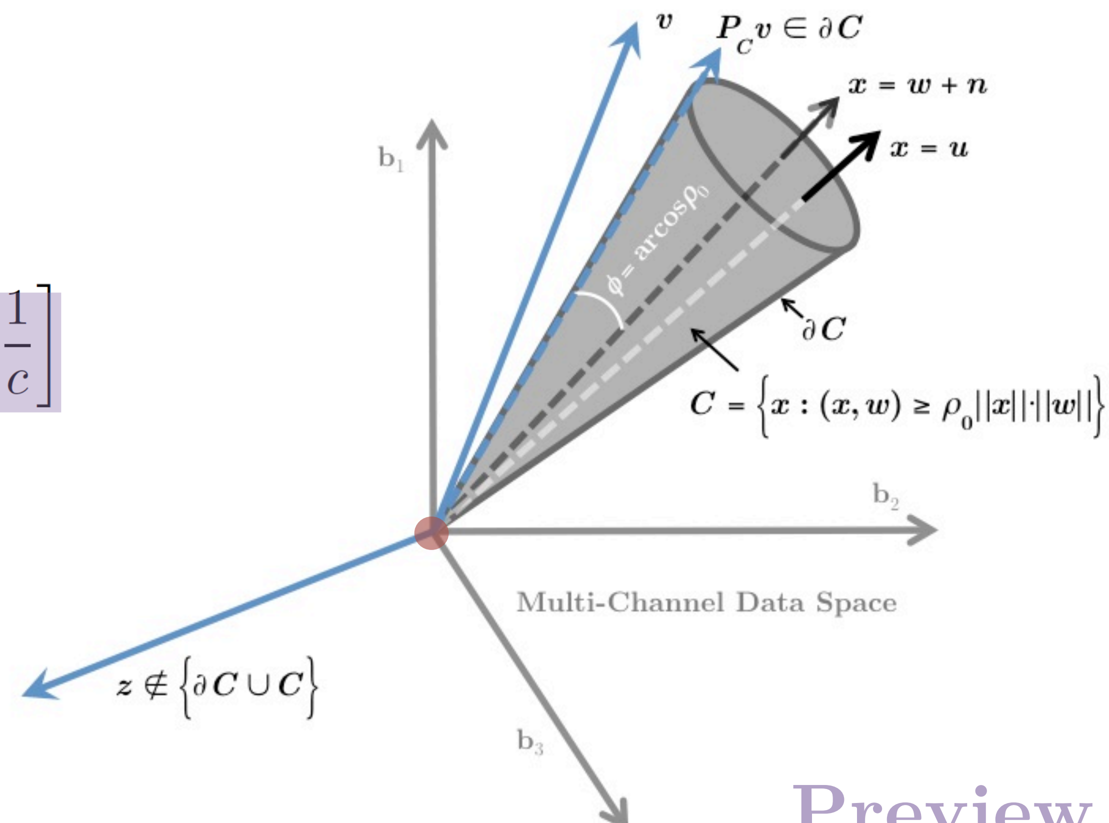
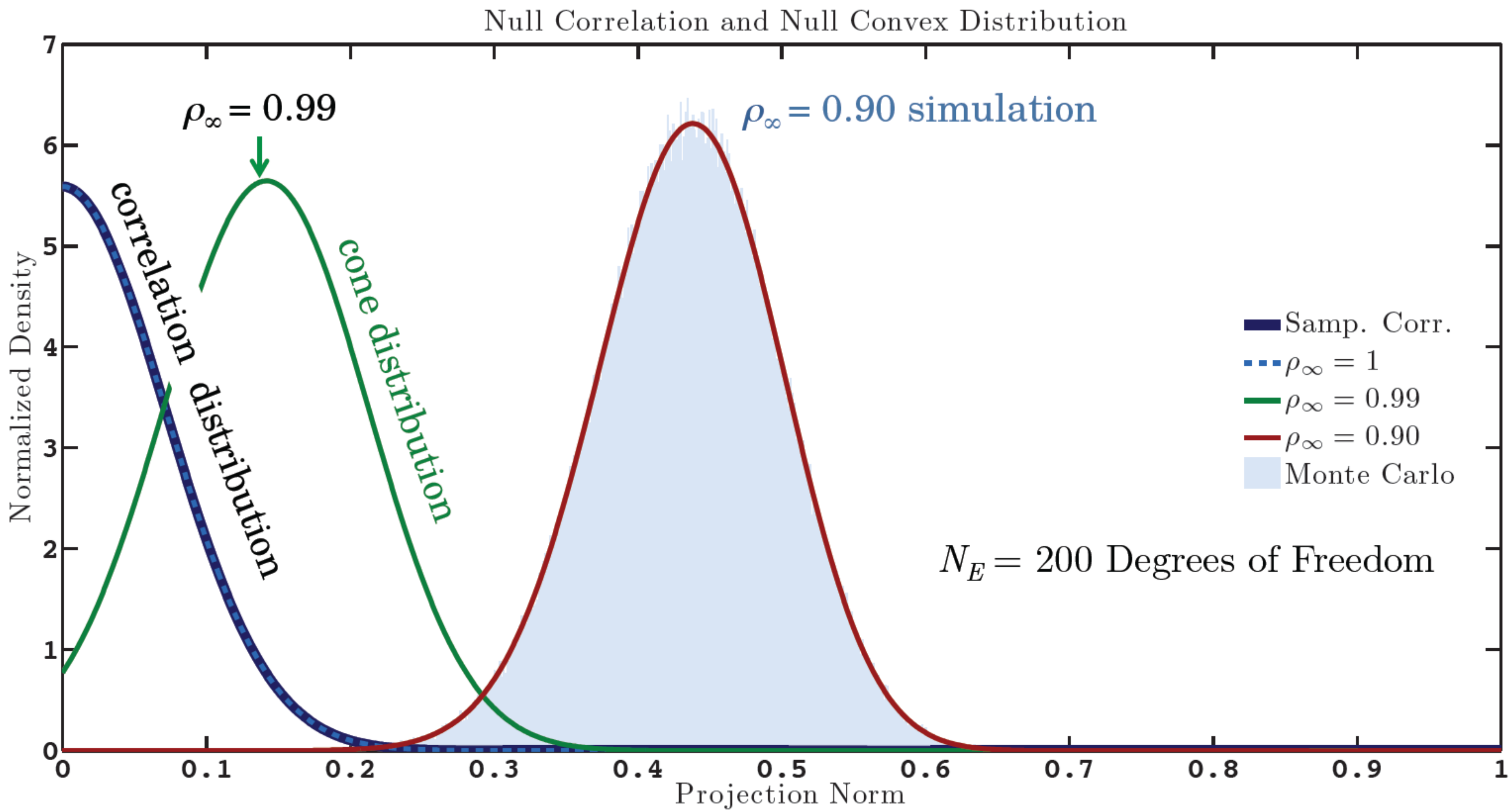


Cone-Detector: Enormous False Alarms

Q: What's the Effect of Uncertainties in template-target on correlation?

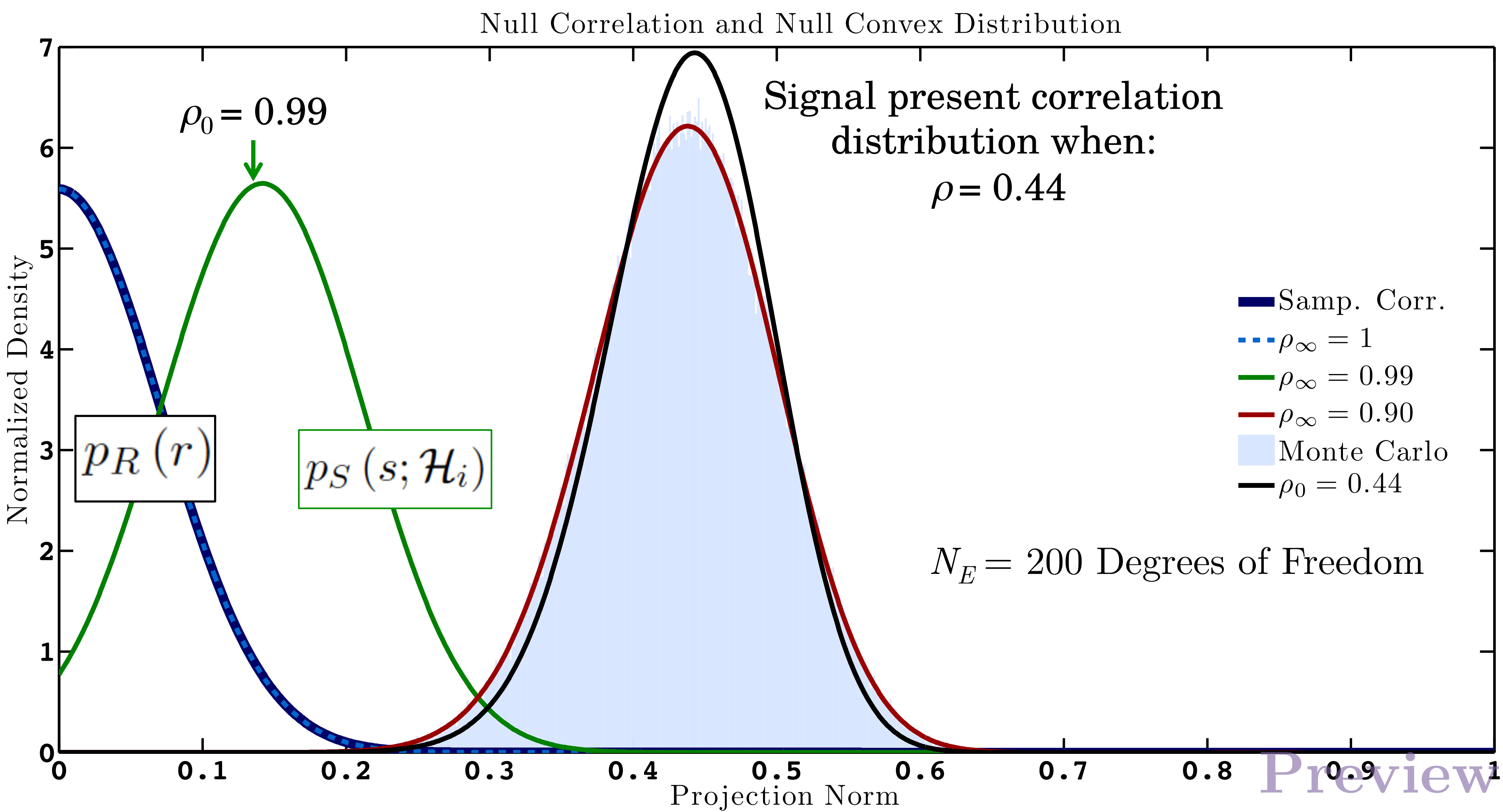
Null distribution is computed from known correlation distribution using variable transformation. It shows probability density of noise giving a detection if the template waveform includes uncertainty



Correlation Detector Sensitivity

Q: How do we include unknown non-target signals in the background wavefield?

Low SNR signal and Null Distributions Overlap if ρ_0 is sufficiently small



Non-Target Signals Live in a Cone

Q: How do we include unknown non-target signals in the background wavefield?

Competing hypotheses

New Null ▶ $\mathcal{H}_0 : \quad \boldsymbol{x} = \boldsymbol{n} + \boldsymbol{u} \sim \mathcal{N}\left(\boldsymbol{u}, \sigma^2 \boldsymbol{I}\right), \quad \boldsymbol{u} \in \mathcal{C}$

Same Altern. $\mathcal{H}_1 : \quad \boldsymbol{x} = \boldsymbol{n} + A\boldsymbol{w} \sim \mathcal{N}\left(A\boldsymbol{w}, \sigma^2 \boldsymbol{I}\right).$

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Same Altern. $\mathcal{H}_1 : \quad \boldsymbol{x} = \boldsymbol{n} + A\boldsymbol{w} \sim \mathcal{N}\left(A\boldsymbol{w}, \sigma^2 \boldsymbol{I}\right).$

Generalized Log-likelihood Ratio

$$\frac{2}{N_E} \ln\left(\Lambda\right) = \ln\left[1 - \frac{\left|\left|P_{\mathcal{C}}\left(\boldsymbol{x}\right)\right|\right|^2 - \left|\left|P_{\boldsymbol{w}}\left(\boldsymbol{x}\right)\right|\right|^2}{\left|\left|P_{\boldsymbol{w}}^{\perp}\left(\boldsymbol{x}\right)\right|\right|^2}\right]$$

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Note argument of scaled log-likelihood is simple

$$\frac{2}{N_E} \ln(\Lambda) = \ln\left[1 - \frac{||P_{\mathcal{C}}(\boldsymbol{x})||^2 - ||P_{\boldsymbol{w}}(\boldsymbol{x})||^2}{||P_{\boldsymbol{w}}^{\perp}(\boldsymbol{x})||^2}\right]$$

Statistic represents difference in projected signal energy: cone – correlation

Screens Targets from Non-Targets

National Nuclear Security Administration

Los Alamos NATIONAL LABORATORY EST.1943

Q: How do we include unknown non-target signals in the background wavefield?

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Statistic represents difference in projected signal energy: cone – correlation

$$\frac{s^2(\boldsymbol{x}) - r^2}{1 - r^2} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

Decision Rule

Screens Targets from Non-Targets

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$$\mathcal{H}_0: \quad \boldsymbol{x} = \boldsymbol{n} + \boldsymbol{u} \sim \mathcal{N}\left(\boldsymbol{u}, \sigma^2 \boldsymbol{I}\right), \quad \boldsymbol{u} \in \mathcal{C}$$

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PDF for statistic has closed form expression:
no Monte Carlo needed to get performance

New Detector: $$\frac{s^2(\boldsymbol{x}) - r^2}{1 - r^2} \underset{\mathcal{H}_0}{\overset{\mathcal{H}_1}{\gtrless}} \eta$$

New Detection Statistic PDF

Q: How do we include unknown non-target signals in the background wavefield?

- Express the detection statistic as a polynomial in $t(r)$:

$$z = \frac{s^2(\boldsymbol{x}) - r^2}{1 - r^2} \triangleq -\left(1-\rho^2\right)t^2 + \left(2\rho\sqrt{1-\rho^2}\right)t + \left(1-\rho^2\right)$$

$$t \triangleq \frac{r}{\sqrt{1-r^2}} \qquad t \text{ has \underline{known} PDF } p_T\left(t\,;\,\mathcal{H}_k\right)$$

- Variable transformation gives point-wise equivalent event:

$$t_{[1]}^{-1}(z) = \frac{1 - \sqrt{1 + c^2\left(1 - \frac{z}{\rho_0 c^2}\right)}}{c}$$

- Get PDF for z:

$$\boxed{p_Z\left(z\,;\mathcal{H}_k\right) = p_T\left(t_{[1]}^{-1}(z)\,;\mathcal{H}_k\right)\left|\frac{dt_{[1]}^{-1}(z)}{dz}\right|}$$

New Detection Statistic PDF

National Nuclear Security Administration

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$$t_{[1]}^{-1}(z) = \frac{1 - \sqrt{1 + c^2\left(1 - \frac{z}{\rho_0 c^2}\right)}}{c}$$

- Get PDF for z: The PDF under $\mathcal{H}_0$ sets detector threshold

$$\boxed{p_Z\left(z\,;\mathcal{H}_k\right) = p_T\left(t_{[1]}^{-1}(z)\,;\mathcal{H}_k\right)\left|\frac{dt_{[1]}^{-1}(z)}{dz}\right|}$$

New Detection Statistic PDF

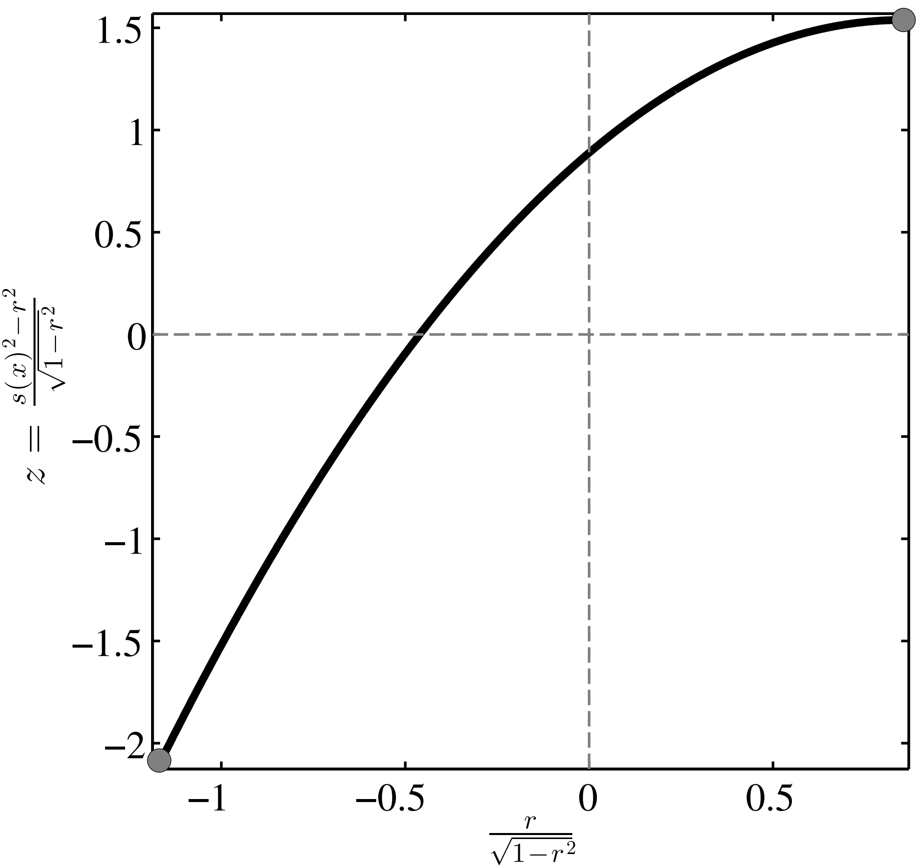
Q: How do we include unknown non-target signals in the background wavefield?

- Luckily, z is one-to-one over our domain...

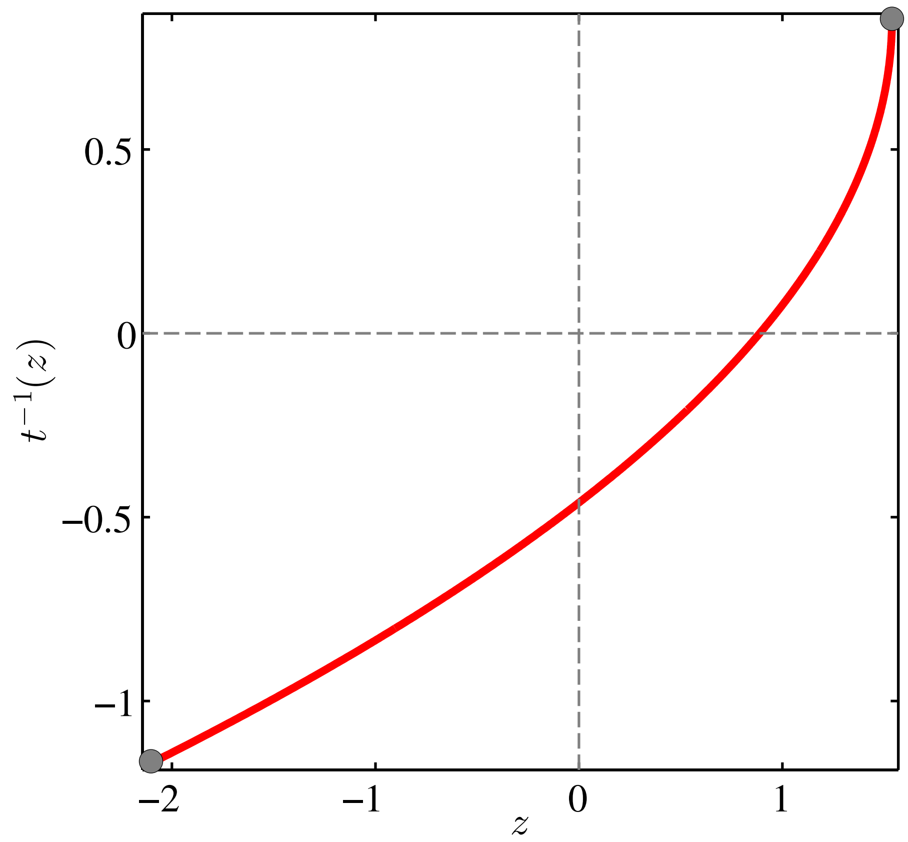
$$z = \frac{s^2(\boldsymbol{x}) - r^2}{1 - r^2}$$

$$t_{[1]}^{-1}(z) = \frac{1 - \sqrt{1 + c^2\left(1 - \frac{z}{\rho_0 c^2}\right)}}{c}$$

Detection Statistic with Convex Null



Inverse Function



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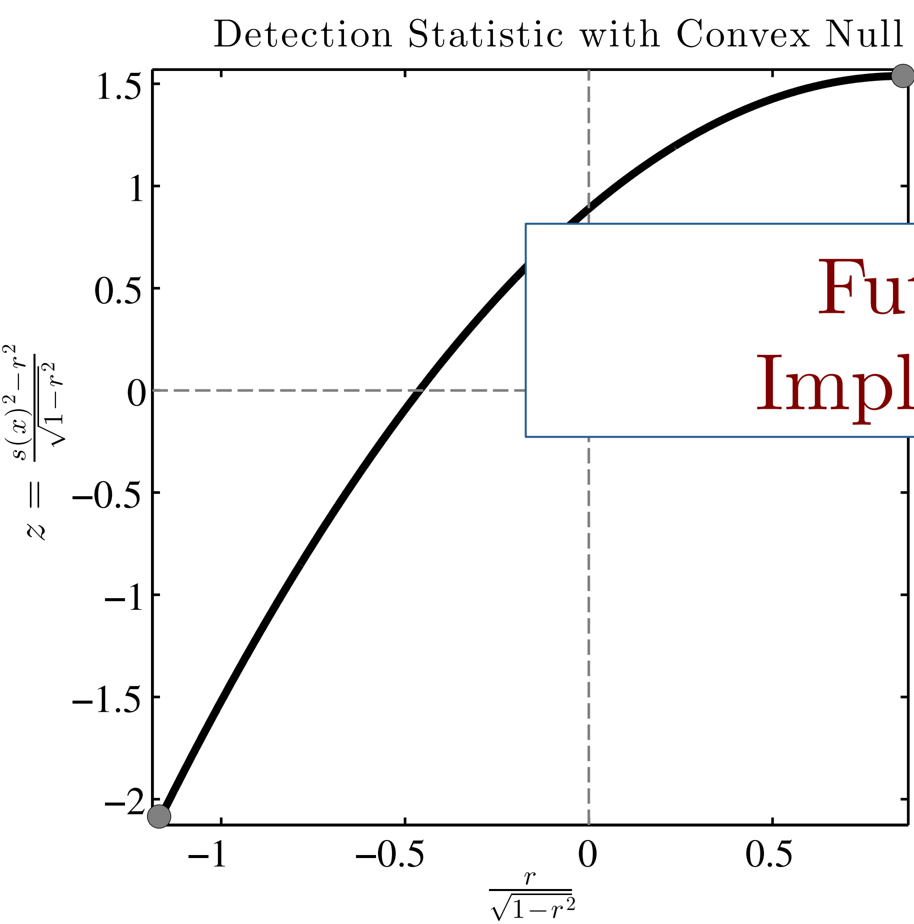

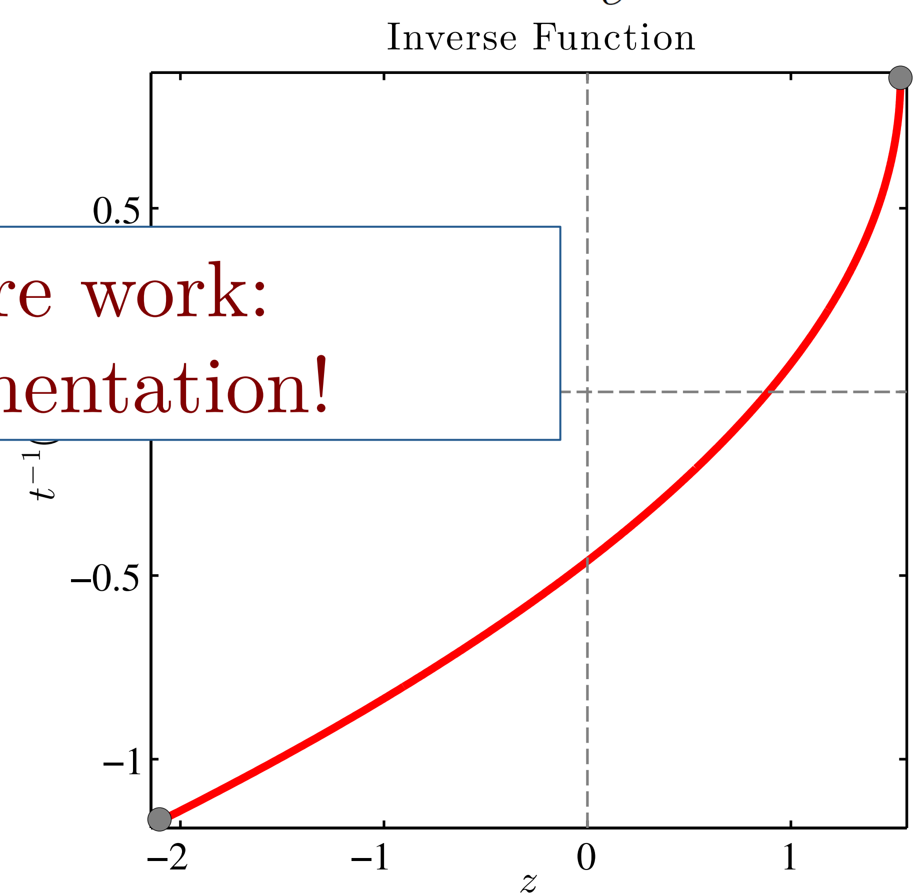

Future work: Implementation!

Review Synthesis

Important Points

- Overwhelming non-target detections require more representative hypothesis test to target real events.
- **Implicit** signal model with convex-cone geometry includes both target waveforms and non-targets
- New detector screens target waveforms from convex cone members that correlation detectors return

Convex Detection

- Proto-type detector returns analyst-equivalent detections
- Requires minimal modification from correlation detector, and has quantifiable detection performance
- Satisfies all solution requirements to reduce false detections for GNDD

References and Credit

References

- Weichecki-Vergara, S., H. L. Gray, and W. A. Woodward (2001), Statistical development in support of CTBT monitoring, Tech. Rep. DTRA-TR-00-22, Southern Methodist Univ., Dallas, Tex.
- Kay, Fundamentals of Statistical Signal Processing: Detection Theory. Englewood Cliffs, NJ: Prentice-Hall, 1998.
- Harris, D. B. (2006), Subspace detectors: theory, Lawrence Livermore National Laboratory Technical Report UCRL-TR-222758, 46 pages, Livermore, CA

Data Sources/Figures

- Plots: figures were generated using MATLAB
- Data: IMS arrays included NVAR and USRK array, OUO data

Personal Communication

- Steven Gibbons: (Sept. 7, 2012) On the variability of effective degrees of freedom of network-observations of signals
- Hans Hartse: (Dec-Jan 2013-2014) Guidance on data aquisition
- David Harris: (July 3, 2012) On usage of the Weichecki-Vergara dimension estimator for dimensionality of random processes