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Title: Extending the radial diffusion model of Falthammar to non-dipole background field

Author(s): Cunningham, Gregory Scott

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Extending the radial diffusion model of Falthammar to non-dipole background field models

GS Cunningham

Space Science and Applications Group, ISR-1, LANL

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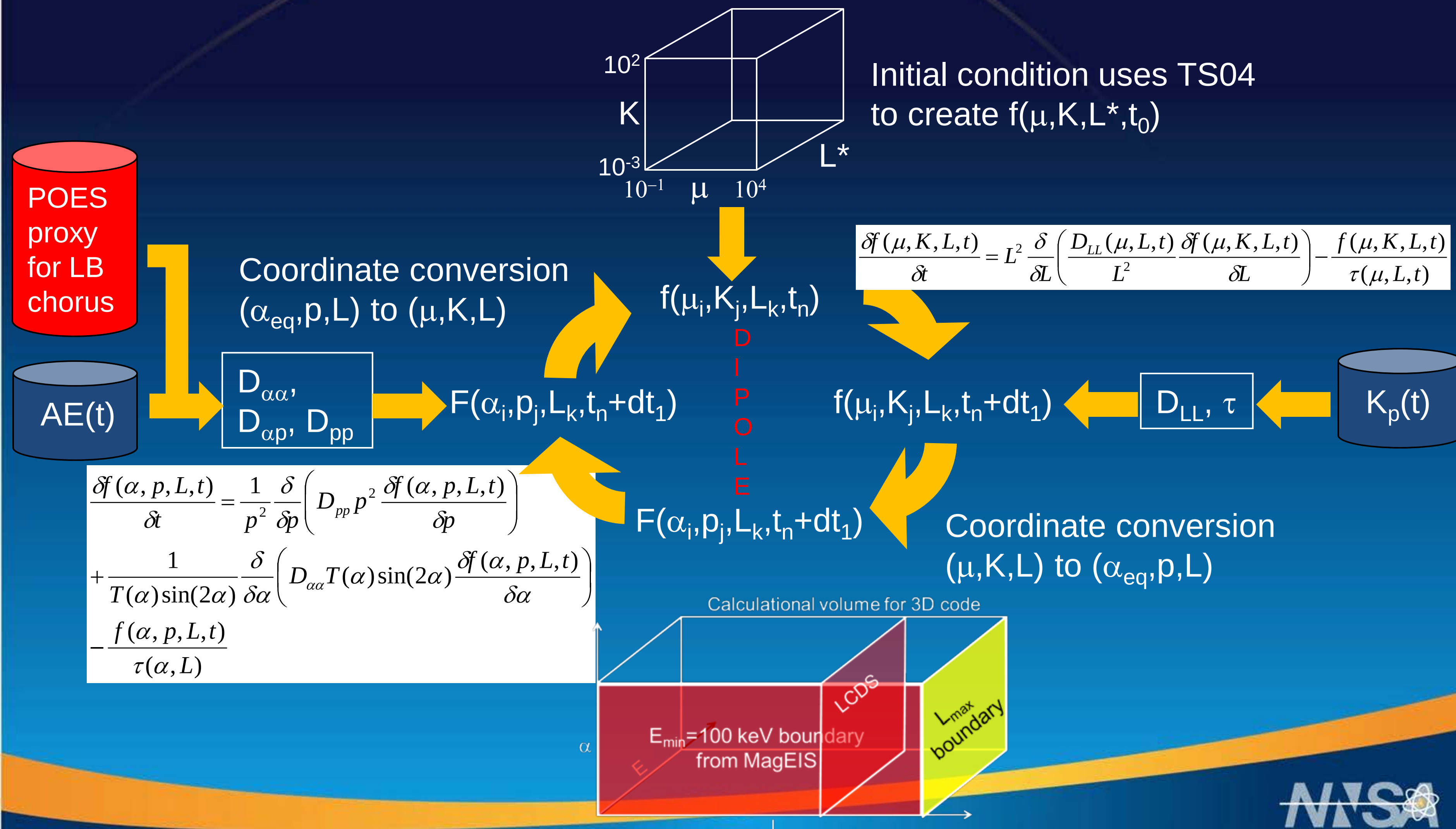
Abstract

A model for radial diffusion caused by electromagnetic disturbances was published by Falthammar (1965) using a two-parameter model of the disturbance perturbing a background dipole magnetic field. Schulz and Lanzerotti (1974) extended this model by recognizing the two parameter perturbation as the leading (non-dipole) terms of the Mead-Williams magnetic field model. They emphasized that the magnetic perturbation in such a model induces an electric field that can be calculated from the motion of field lines on which the particles are 'frozen'. Roederer and Zhang (2014) describe how the field lines on which the particles are frozen can be calculated by tracing the unperturbed field lines from the minimum-B location to the ionospheric footpoint, and then tracing the perturbed field (which shares the same ionospheric footpoint due to the frozen-in condition) from the ionospheric footpoint back to a perturbed minimum B location. The instantaneous change in Roederer L*, dL*/dt, can then be computed as the product (dL*/dphi)*(dphi/dt). dL*/dphi is linearly dependent on the perturbation parameters (to first order) and is obtained by computing the drift across L*-labeled perturbed field lines, while dphi/dt is related to the bounce-averaged gradient-curvature drift velocity. The advantage of assuming a dipole background magnetic field, as in these previous studies, is that the instantaneous dL*/dt can be computed analytically (with some approximations), as can the DLL that results from integrating dL*/dt over time and computing the expected value of (dL*)^2. The approach can also be applied to complex background background magnetic field models like T89 or TS04, on top of which the small perturbations are added, but an analytical solution is not possible and so a numerical solution must be implemented. In this talk, I discuss our progress in implementing a numerical solution to the calculation of DL*L* using arbitrary background field models with simple electromagnetic perturbations.

DREAM3D is a 3D diffusion code implemented as 1D+2D diffusion with event-specific chorus wave amplitude, E_{min} boundary condition, and LCDS

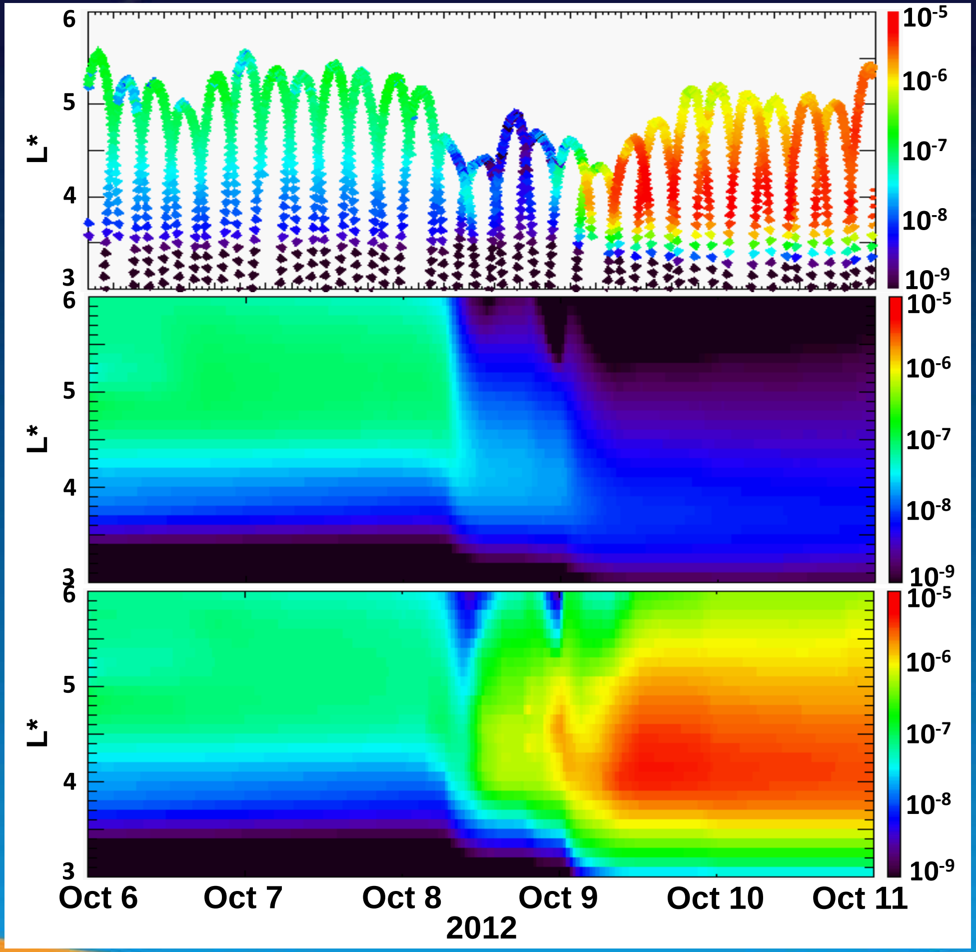


$$\frac{\delta f(\mu,K,L,t)}{\delta t} = L^2 \frac{\delta}{\delta L}\left(\frac{D_{LL}(\mu,L,t)}{L^2}\frac{\delta f(\mu,K,L,t)}{\delta L}\right) - \frac{f(\mu,K,L,t)}{\tau(\mu,L,t)}$$

$$\frac{\delta f(\alpha,p,L,t)}{\delta t} = \frac{1}{p^2}\frac{\delta}{\delta p}\left(D_{pp}p^2\frac{\delta f(\alpha,p,L,t)}{\delta p}\right) + \frac{1}{T(\alpha)\sin(2\alpha)}\frac{\delta}{\delta \alpha}\left(D_{\alpha\alpha}T(\alpha)\sin(2\alpha)\frac{\delta f(\alpha,p,L,t)}{\delta \alpha}\right) - \frac{f(\alpha,p,L,t)}{\tau(\alpha,L)}$$

DREAM3D reproduces enhancement but not dropout (Tu et al GRL 2014)

PSD[μ=1279 MeV/G, K=0.115 $G^{1/2}R_E$,L*,t]



Van Allen Probes electron data (MagEIS, REPT)

DREAM3D w/ event-specific E_{min} BC, LCDS, but **w/o** event-specific lowband chorus wave intensity

DREAM3D w/ event-specific E_{min} BC, LCDS, and **with** event-specific lowband chorus wave intensity

(A few of the) limitations of previous work

- VLF wave-particle interactions
  - Plasma trough density observed by EMFISIS lower than Sheeley trough model used in DREAM3D; will cause additional acceleration
  - Magnetic field intensity observed by EMFISIS significantly different from dipole; will affect acceleration, loss, and drift orbits
- ULF wave-particle interactions
  - Magnetic field intensity observed by EMFISIS significantly different from dipole; will affect drift orbits
- Goal of current work: incorporate non-dipole field models in DREAM3D to enable use of calculated drift shells and field intensity, enabling more accurate modeling of ULF/VLF wave-particle interactions

Using the T89/TS04 empirical field model in DREAM3D: background

- DREAM3D uses the (μ,K,L*) invariants

1st

$$\mu = \frac{p_\perp^2}{2m_0 B}$$

2nd

$$J = 2\int_{s_m}^{n_m} p_\parallel ds = 2pI$$

$$I = \int_{s_m}^{n_m} \sqrt{1-\left(\frac{B(s)}{B_m}\right)} ds$$

$$K = \sqrt{B_m} I$$

3rd

$$L^* = \frac{2\pi B_E R_E^2}{\Phi}$$

- Geometric sampling in μ,K; uniform in L*

$$\mu_i = 10^{-2}\left(\frac{10^5}{10^{-2}}\right)^{(i+0.5)/200}$$

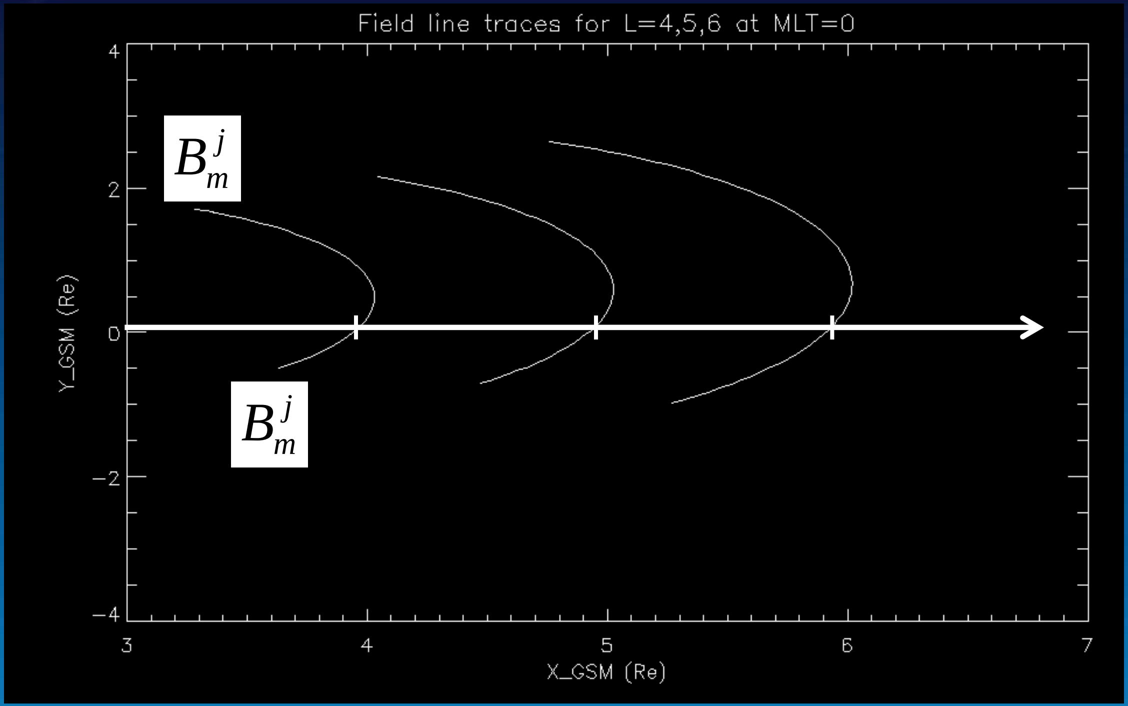
$$K_j = 10^{-3}\left(\frac{10^1}{10^{-3}}\right)^{(j+0.5)/100}$$

$$L_k^* = 1 + (k+0.5) * 0.1$$

Using the T89/TS04 empirical field model in DREAM3D

- For each time step in DREAM3D (dt=1/2 hour for outer-belt study),
  - For each dL=0.1 R_e along a ray at 00 MLT
    - For each K_j
      - Trace drift shell to give (x,y,z,B) along the field line as a function of MLT
      - Compute L* for this drift shell
  - Rebin the drift-shells uniformly in L* so that one has L(MLT, K, L*) for each L* bin in the calculation
  - Use the drift-shell to compute drift-averaged quantitites

Computing the drift shells: trace initial field lines

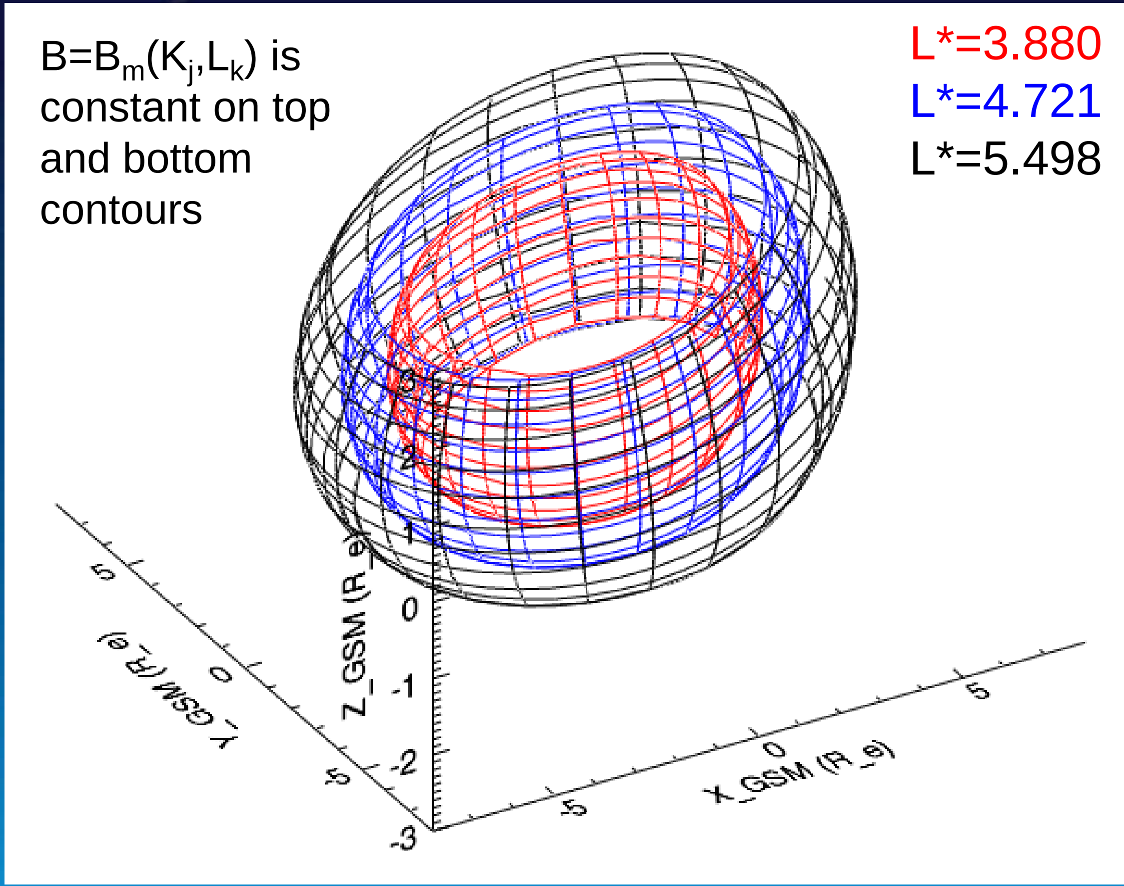


1. Trace field line to 100 km for each L_k=1.0+(0.1*(k+0.5)) along a ray pointed to midnight.

2. For each field line, determine the mirror-point magnetic field $B_m(K_j)$ for each K_j on the desired grid.

3. Save the portion of the field line that mirrors at $B_m(K_j)$ in the north and south.

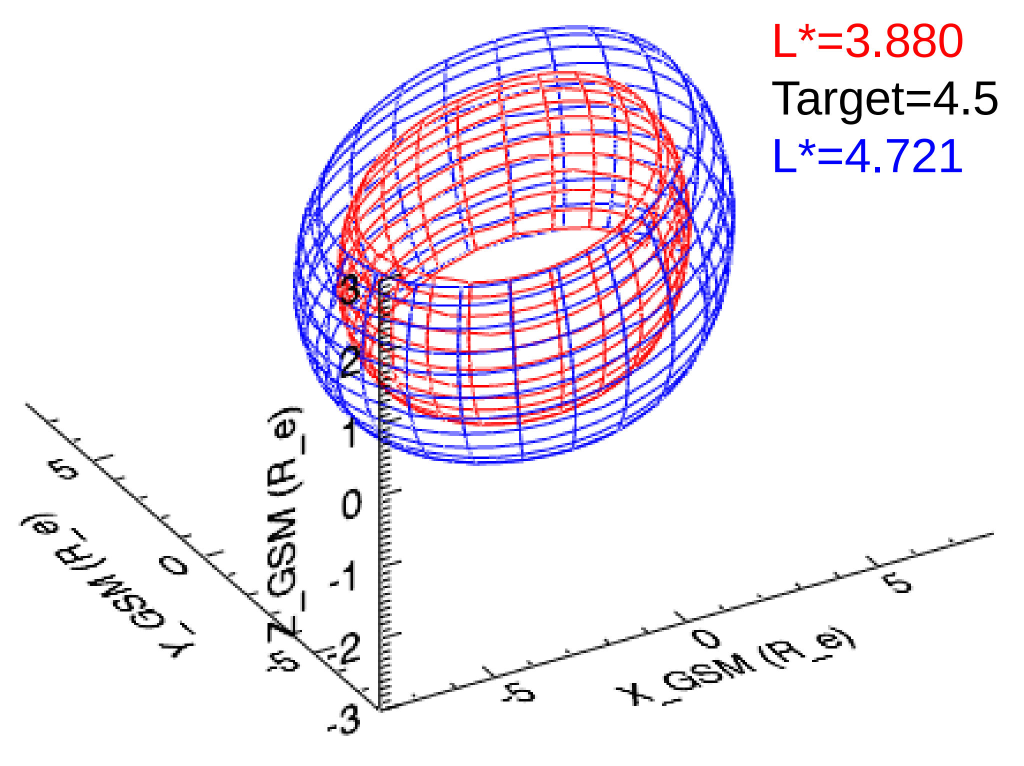
Computing the drift shells: trace shell at constant K, B_m



1. For each (L_k, K_j) trace out the drift shell by finding the field line at each MLT that has the same (K_j, $B_m(K_j,L_k)$). A drift shell is specified by ($B(s,\phi)$, $x(s,\phi)$, $y(s,\phi)$, $z(s,\phi)$).

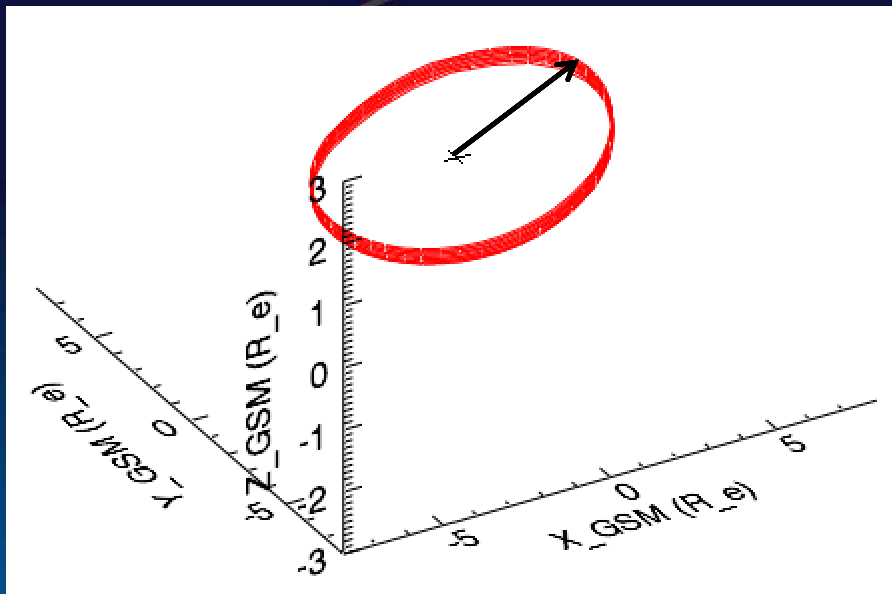
2. Compute the Roederer L* for each drift shell. These L* values are not uniformly spaced.

Computing the drift shells: resample onto uniform L*

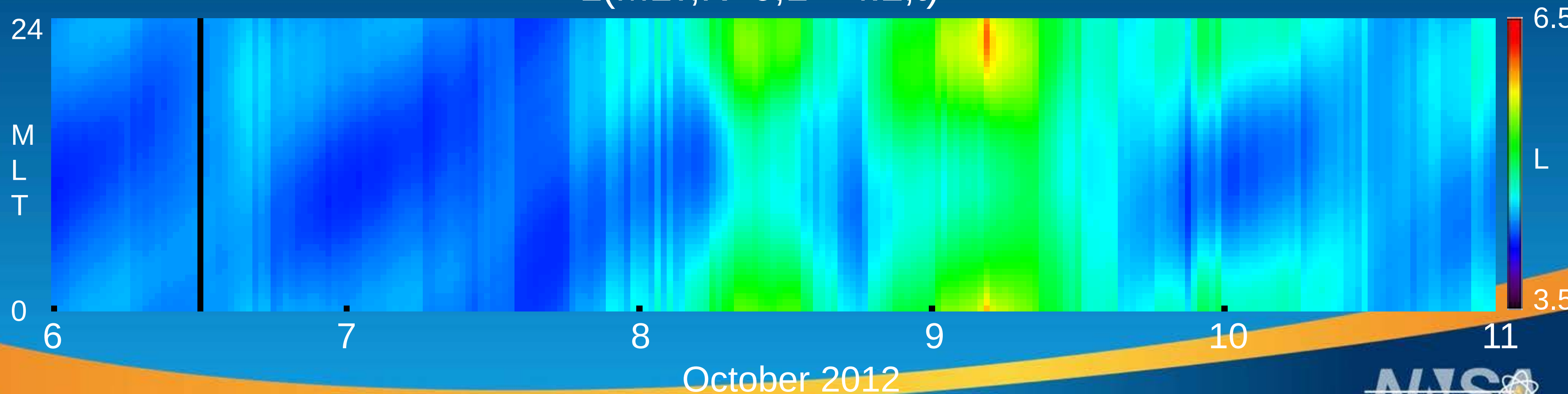


1. For each target L^*_k on the uniformly sampled grid in L*, and for each K_j, find the two drift shells that bracket the target L*.
2. Interpolate the two drift shells at each (s,ϕ) using $(B_1(s,\phi),x_1(s,\phi),y_1(s,\phi),z_1(s,\phi))$ $(B_2(s,\phi),x_2(s,\phi),y_2(s,\phi),z_2(s,\phi))$
3. Now have a drift shell for each target grid point K_j,L^*_k

Drift shell inflation during a storm



For each ½ hour time interval, for the K~0 drift shell, find the distance to B_{min}, LR_e, for each MLT. Call this L(MLT,K~0,t) and plot below.



Naïve use of TS04 drift shells for radial diffusion makes substantial difference

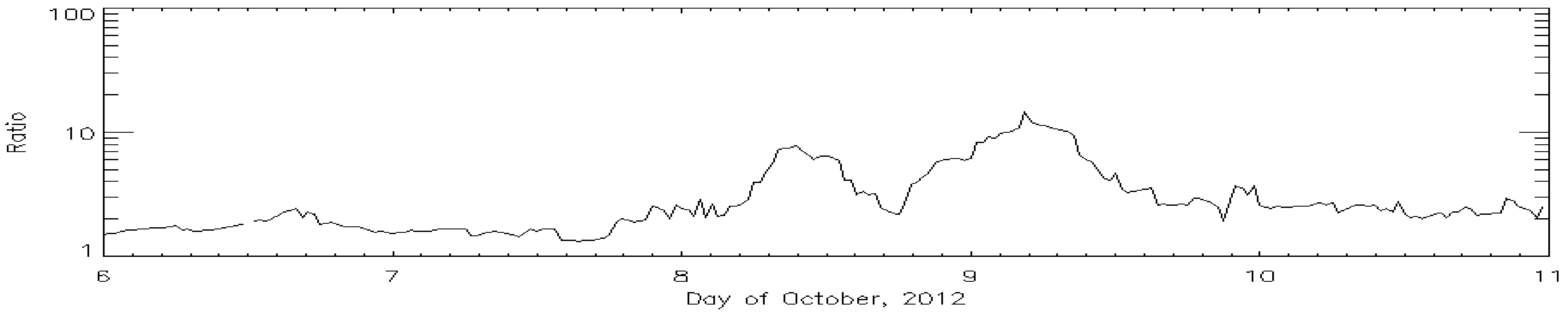
- DREAM3D old: assume L=L*

$$D_{LL} = D_{LL}^{0}\left(K_p\right)\left(L^*\right)^{10}$$

- DREAM3D new: average over MLT

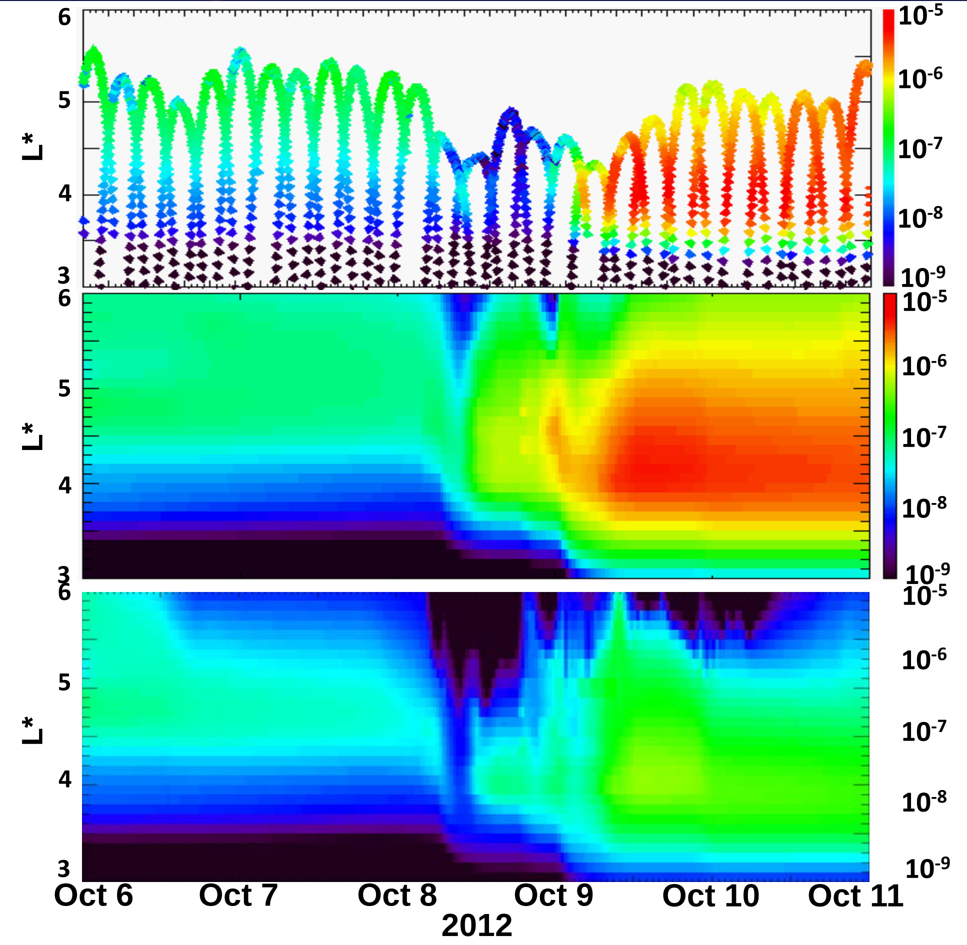
$$D_{LL} = \frac{1}{N_{MLT}} \sum_{n=0}^{N_{MLT}-1} D_{LL}^{0}\left(K_p\right)\left(L\left(MLT_n, K_j, L_k^*\right)\right)^{10}$$

Ratio of new to old at L*=4.2 vs time



Naïve use of drift-shells for radial diffusion has significant impact on DREAM3D output

PSD[μ=1279 MeV/G, K=0.115 $G^{1/2}R_E$,L*,t]



Van Allen Probes electron data (MagEIS, REPT)

DREAM3D PSD using old D_{LL}(L=L*)

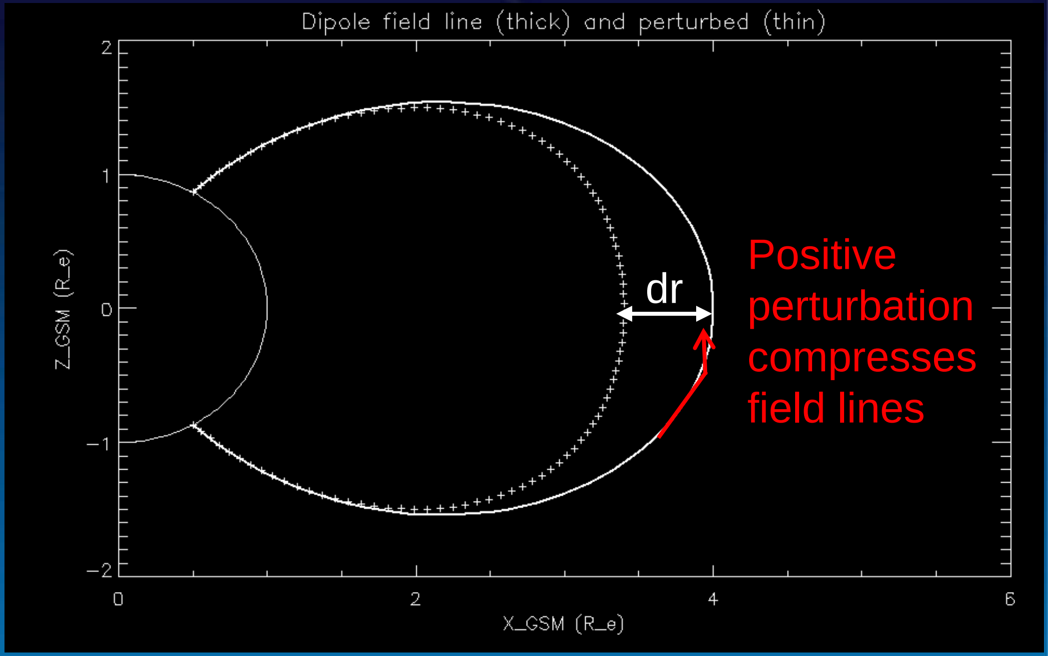
DREAM3D PSD using MLT-averaged D_{LL}(L(MLT,K,L*))

Naïve use of drift-shells for MLT-averaged D_{LL} inadequate

- Falthammar diffusion from magnetic perturbations
  - $\langle(\Delta r)^2\rangle \propto \frac{r^4\Omega^2}{B^2} S(\Omega) \propto r^{10}$ if S(f) $\propto \frac{1}{f^2}$
- We're simply using inflated drift-shell radii for r in the formula above, but the derivation depends on many aspects of dipole field field
  - Constant drift velocity, Ω
  - Circular drift orbits
  - Magnetic field intensity dependence on r
  - L*=L
- Need to start from scratch for non-dipole background field

Start with Schulz and Lanzerotti 1974 derivation for perturbations to a dipole

- Magnetic perturbation A(t)**b**(**x**) causes the field lines on which particles are 'frozen' to move
- Ionospheric footpoints of perturbed field lines are unchanged
- Field line movement induces an electric field $v_d = ExB/|B|^2$



Computation of unperturbed field line

$$\frac{dr}{rd\theta} = \frac{d}{d\theta}\ln(r(\theta)) = \frac{B_r}{B_\theta}$$

$$\ln(r(\theta)) = \int \frac{2\cos(\theta')}{\sin(\theta')} d\theta' + L = \ln\left(\sin^2(\theta)\right) + L$$

$$r(\theta) = L\sin^2(\theta)$$

Computation of perturbed field line

$$\frac{d}{d\theta}\ln\left(r^{(1)}(\theta)\right)=\frac{B_r\left(r^{(0)}(\theta),\theta\right)+A(t)b_r\left(r^{(0)}(\theta),\theta\right)}{B_\theta\left(r^{(0)}(\theta),\theta\right)+A(t)b_\theta\left(r^{(0)}(\theta),\theta\right)}$$

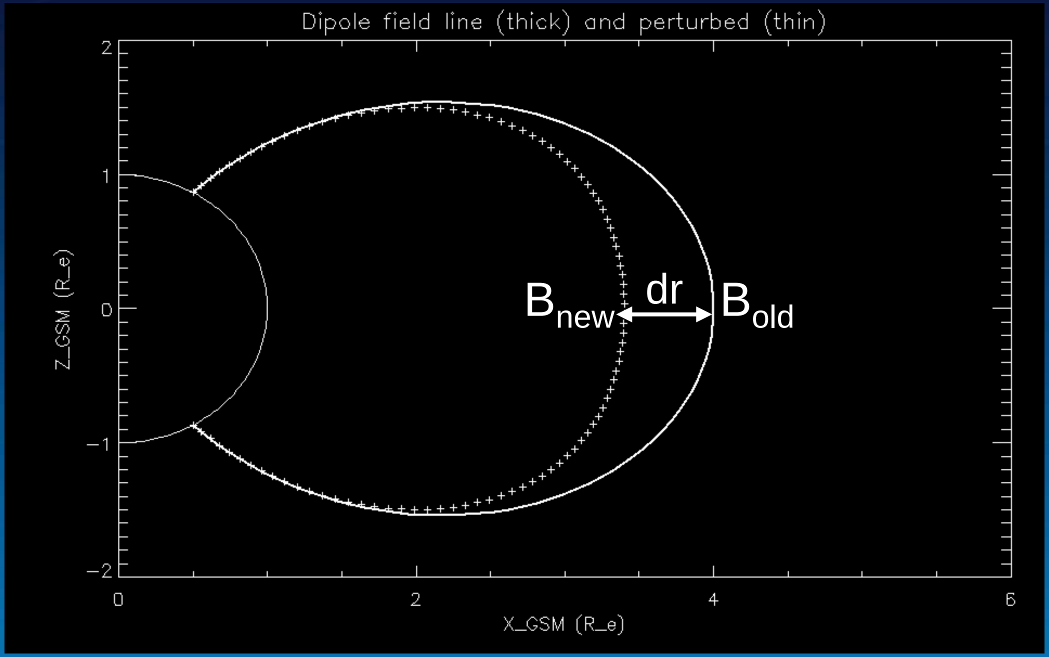
Drop 2nd order terms

$$\ln\left(r^{(1)}(\theta)\right)\sim\int\frac{B_r\left(r^{(0)}(\theta'),\theta'\right)}{B_\theta\left(r^{(0)}(\theta'),\theta'\right)}\left[1+A(t)\frac{b_r\left(r^{(0)}(\theta'),\theta'\right)}{B_r\left(r^{(0)}(\theta'),\theta'\right)}-A(t)\frac{b_\theta\left(r^{(0)}(\theta'),\theta'\right)}{B_\theta\left(r^{(0)}(\theta'),\theta'\right)}\right]$$

$$r^{(1)}(\theta)=r^{(0)}(\theta')\left[1+A(t)\left\{\int\frac{b_r\left(r^{(0)}(\theta'),\theta'\right)}{B_\theta\left(r^{(0)}(\theta'),\theta'\right)}d\theta'-\int\frac{B_r\left(r^{(0)}(\theta'),\theta'\right)b_\theta\left(r^{(0)}(\theta'),\theta'\right)}{\left(B_\theta\left(r^{(0)}(\theta'),\theta'\right)\right)^2}d\theta'\right\}\right]$$

$$dr(\theta)=A(t)r^{(0)}(\theta)\int\left\{\frac{b_r\left(r^{(0)}(\theta'),\theta'\right)}{B_\theta\left(r^{(0)}(\theta'),\theta'\right)}-\frac{B_r\left(r^{(0)}(\theta'),\theta'\right)b_\theta\left(r^{(0)}(\theta'),\theta'\right)}{\left(B_\theta\left(r^{(0)}(\theta'),\theta'\right)\right)^2}\right\}d\theta'$$

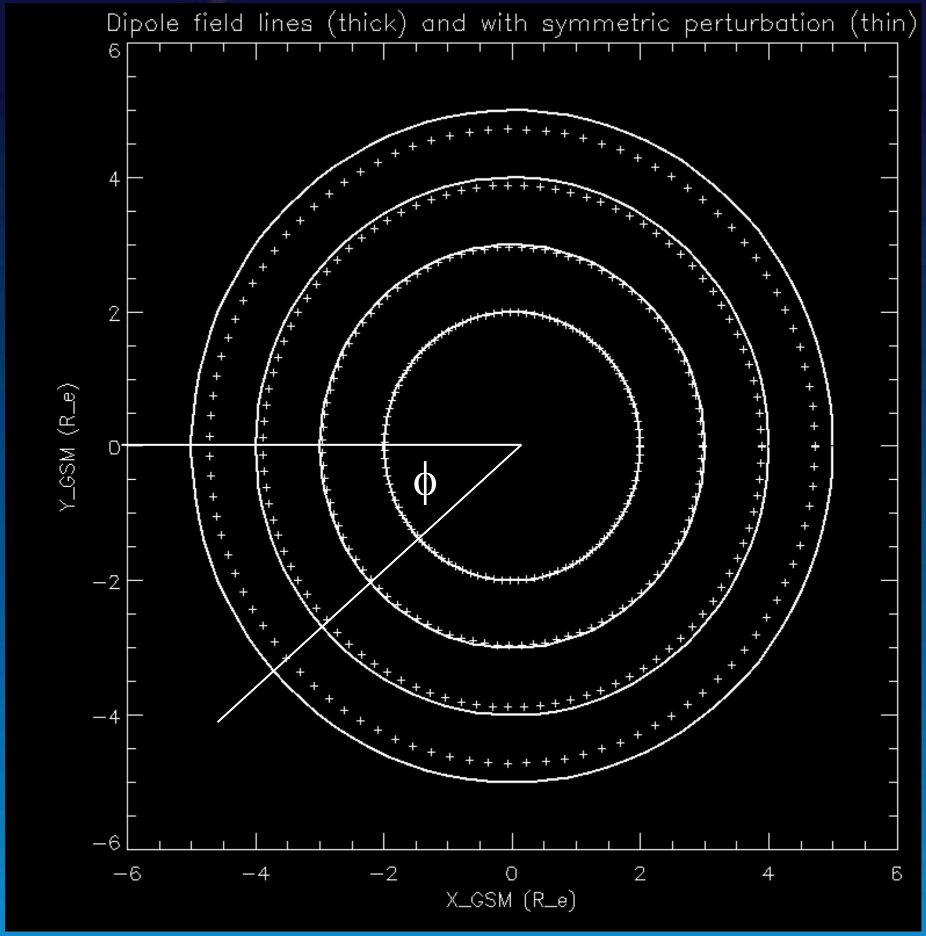
Field lines for 'labeled' drift shells are perturbed, but μ, K are conserved



Invariants μ, K are conserved, but B_m, KE and α are not.

L* label stays the same if particle remains on this field line EVEN THOUGH the instantaneous Roederer L* changes (2^{nd} order?).

Symmetric background field (dipole) and symmetric perturbation imply no diffusion

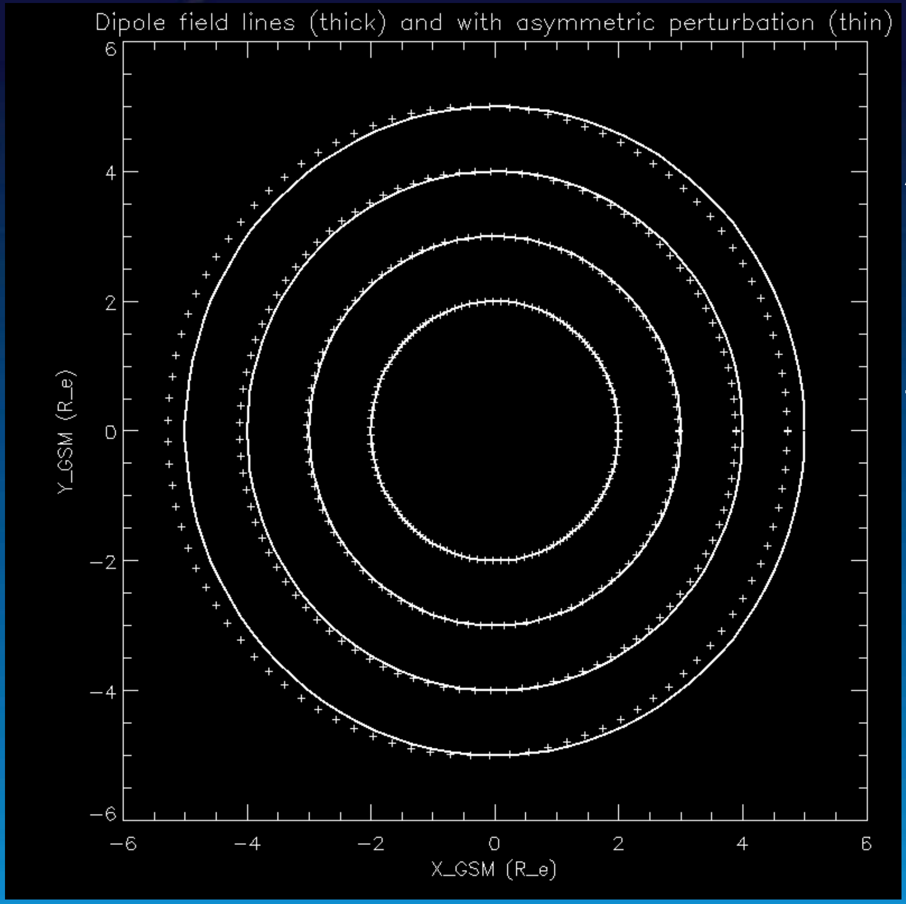


Symmetric perturbation to a symmetric background field compresses field lines at different MLT the same.

Particle drift causes particles to move onto field lines at different MLT with same L* label since $dr(\phi)=dr$.

Since L*(t) is constant, no radial diffusion.

Particle drift in MLT and asymmetry needed to change L* 'label'



Asymmetric perturbation compresses field lines at different MLT differently, dr(ϕ).

Particle drift causes particles to move onto field lines at different MLT with different L* labels.

Since L*(t) is not constant, potential for radial diffusion.

$$\langle V_d \rangle = \frac{2\left(\frac{\gamma+1}{\gamma}\right)KE}{qS_b\left|\vec{B}_0\right|^2}\nabla I \times \vec{e}_0$$

Instantaneous change in L* label caused by particle drift and asymmetry

- Drift is perpendicular to gradient of K: conservation of K

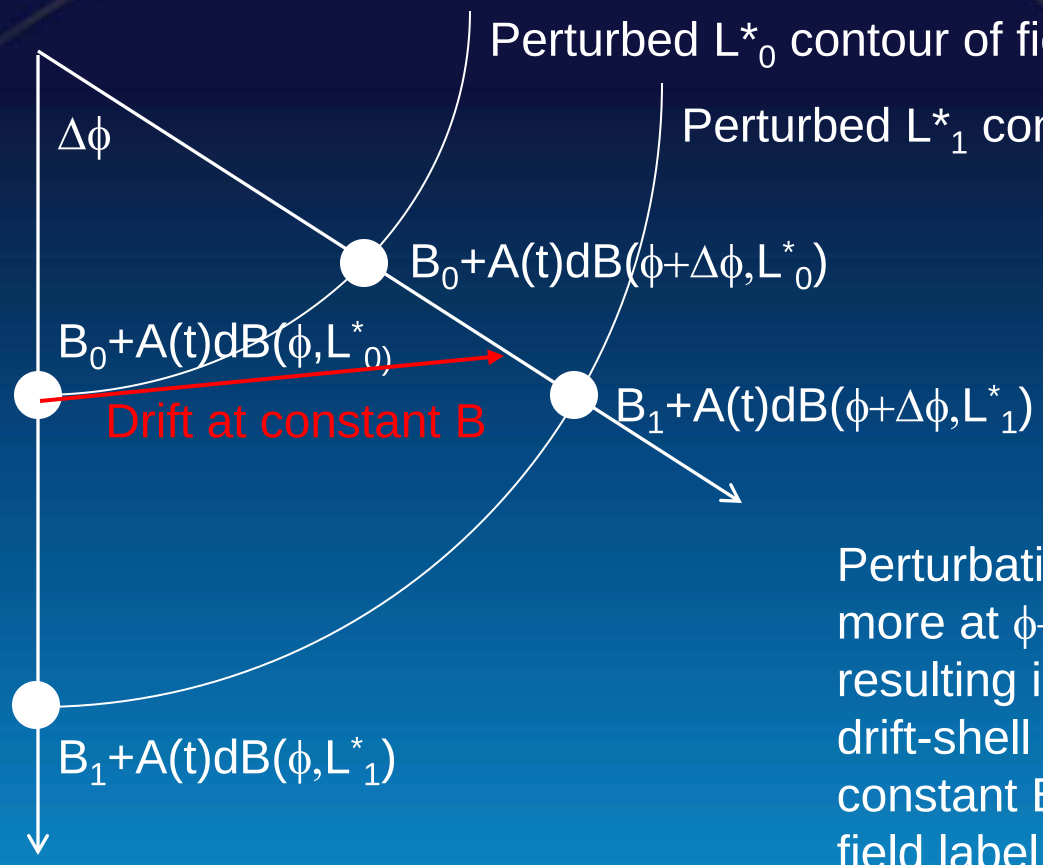
$$\langle V_d \rangle = \frac{2\left(\frac{\gamma+1}{\gamma}\right)KE}{qS_b\left|\vec{B}_0\right|^2}\nabla I \times \vec{B}_0$$

- Change in L* label also given by conservation of K

$$dK = \left.\frac{\delta K}{\delta L^*}\right|_{B_m}\Delta L^* + \left.\frac{\delta K}{\delta \phi}\right|_{B_m}\Delta\phi = 0, \quad \frac{\Delta L^*}{\Delta \phi} = -\frac{\left.\delta K/\delta\phi\right|_{B_m}}{\left.\delta K/\delta L^*\right|_{B_m}}$$

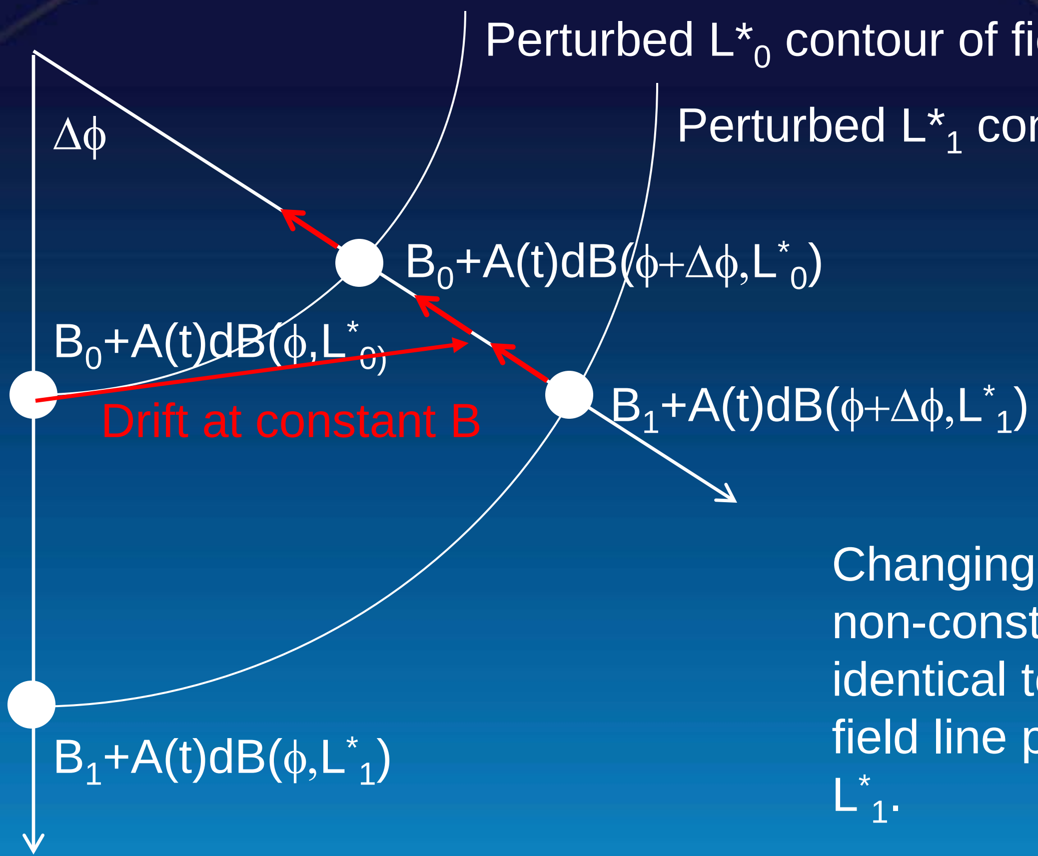
- Time rate of change in L* label: dL*/dt=(dL*/dϕ)(dϕ/dt)

Example dL*/dϕ for K=0



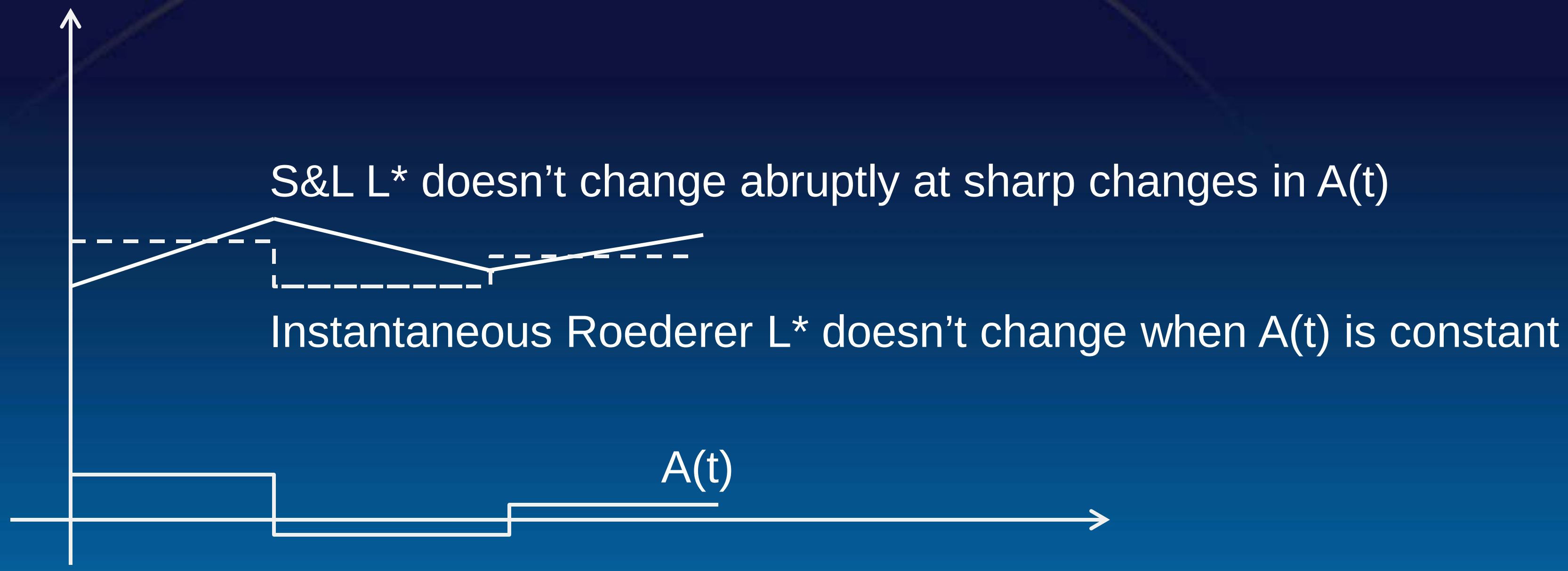
Perturbation compresses field more at $\phi+\Delta\phi$ than at ϕ, resulting in larger B for same drift-shell label, L*. Drift at constant B causes change in field label, hence ΔL^* over $\Delta\phi$.

Example dL*/dϕ: what about changing A(t)?



Changing A(t) causes drift at non-constant B, with the change identical to the change at the field line points labeled L^*_0 and L^*_1.

Schulz and Lanzerotti L* label is not instantaneous Roederer L*



Example for K=0: numerical evaluation of dL*/dϕ

- Given a ‘background’ field, **B**(**x**), for each (K, L*, ϕ)
  - Have field line mapped to ionosphere from drift-shell tracing
- For time-varying perturbation A(t)**b**(**x**)
  - Perform integral that computes perturbation A(t)dr
    - equivalent to mapping field line for **B**(**x**)+A(t)**b**(**x**) from ionospheric footpoint to B_{min} to 1st order
  - Compute dB=A(t){(δ|**B**|/δr)*dr+|$\mathbf{b}_{par}$(**x**)|}
- Using $B_{perturbed}$(K,L*,ϕ)=$B_{unperturbed}$+dB, trace drift at constant B to compute dL*/dϕ=A(t)f(K,L*,ϕ)

Additional work to get $D_{L^*L^*}$

- For a particle starting at MLT (K, L*, ϕ_0)

$$\Delta L^* = \int_0^t \frac{dL^*(K, L^*, \phi(t'))}{dt} dt' = \int_0^t f(K, L^*, \phi(t')) A(t') \Omega_d(\phi(t')) dt'$$

$$\phi(t) = \phi_0 + \int_0^t \Omega_d(\phi(t')) dt'$$

- The drift period, T, where $2\pi = \int_0^T \Omega_d(\phi(t')) dt'$ plays a critical role since

$$g(t) = f(K, L^*, \phi(t)) \Omega_d(\phi(t))$$

is periodic with period T $g(t+T) = g(t)$

Additional work to get $D_{L^*L^*}$

- Since g(t) is periodic we have $g(t)=\sum_{n=-\infty}^{\infty} g_n e^{jn2\pi/T}$
- The diffusion coefficient samples the power spectrum of A(t) at harmonics of the drift frequency

$$D_{L^*L^*} = \lim_{t\to\infty} \frac{E\left[\left(\Delta L^*\right)^2\right]}{2t} = \lim_{t\to\infty} \frac{1}{2t}\int_0^t\int_0^t \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} g_n^* g_m e^{j2\pi m t_1/T} e^{-j2\pi n t_2/T} R_A(t_1-t_2)\,dt_1 dt_2$$

$$= \lim_{t\to\infty} \frac{1}{2t}\int_0^{2t}\int_{-u}^{u} \sum_{m=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} g_n^* g_m e^{j2\pi(m-n)u/2T} e^{j2\pi(m+n)\tau/2T} R_A(\tau)\,d\tau\,du$$

$$= \sum_{m=-\infty}^{\infty} \left|g_m\right|^2 S_A\left(\frac{2\pi m}{T}\right)$$

Summary

- The work of Falthammar and Schulz & Lanzerotti uses 1^{st}-order approximations for the effect of a small, ultra-low frequency electromagnetic perturbation to the magnetic field lines on a labeled drift shell.
- Schulz and Lanzerotti showed that a change in the drift-shell label, L*, can occur when the particle drifts from a field line associated with one label to another, and computed diffusion in the L* label for a dipole background field perturbed by a one-parameter model.
- This approach can be applied to more complicated background field models (T89, TS04) using the same (or different!) perturbations and solved numerically.