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Title: The Role of Scale and Model Bias in ADAPT's Photospheric Eatimation

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The Role of Scale and Model Bias in ADAPT's Photospheric Estimation

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ADAPT model

Air Force Data Assimilative Photospheric flux Transport model

- Magnetic flux propagation based on Worden-Harvey (WH) model
  - **Differential Rotation**
  - **Meridional Flow**
  - **Supergranular Diffusion**
  - **Background Flux Emergence**

Goal: Combine WH model with photosphere observations

A: Provide global photospheric map

B: Enhance Earth side photosphere observation

SOLIS-VSM line-of-sight

Observation & Noise

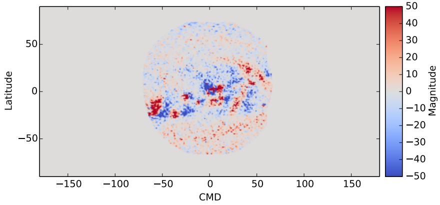


Figure : Synoptic Optical Longterm Investigations of the Sun Vector Spectromagnetograph (SOLIS-VSM)

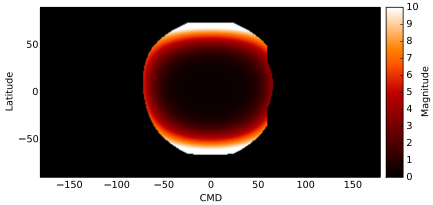
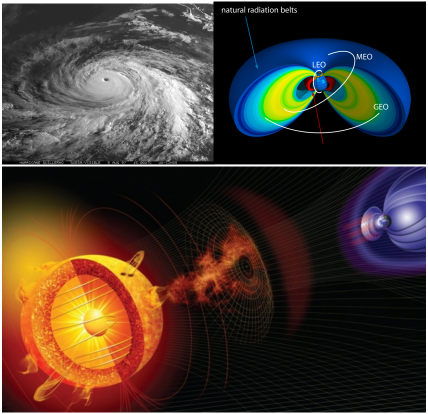


Figure : Estimated observation error/standard deviation. Less certainty at limbs due to line-of-sight observations.

Data Assimilation
General Problem & Notation



Data assimilation: methods that combine information from a model, observational data, and error statistics, to provide an estimate of true state of a system.

- Weather prediction
- Hurricane simulation and forecasting
- Radiation belt simulation
- Solar Physics

The Ensemble Kalman Filter (EnKF) was first introduced by Evensen (1994) as a Monte Carlo approximation to Kalman filtering and has gained wide acceptance in data assimilation applications

Notes/Assumptions:

- EnKF is a sequential data assimilation method that uses an ensemble of model forecast to approximate the model mean and covariance matrix
- Model distribution is Gaussian, we only need the mean and covariance to fully describe the distribution
- Model errors are small compared with errors in initial condition/prior state, and parameters
- Observations can be represented in ensemble of forecast

Let $\mathcal{M}_{t_k \to t_{k+1}}$ be the forecast model,

$$\mathbf{x}(t_{k+1}) = \mathcal{M}_{t_k \to t_{k+1}}(\mathbf{x}(t_k)) \tag{1}$$

For an vector of observations $\mathbf{y}^o \in \mathbb{R}^m$ and an ensemble of N forecast $\mathbf{x}_i^f \in \mathbb{R}^n, i = 1, \ldots, N$ the EnKF analysis equation are given by:

$$\begin{aligned} \mathbf{x}_i^a &= \mathbf{x}_i^f + \mathbf{K}\left(\mathbf{y}_i^o - \mathbf{H}\mathbf{x}_i^f\right), \quad i = 1, \ldots, N & (2) \\ \mathbf{K} &= \mathbf{P}^f \mathbf{H}^T \left(\mathbf{H}\mathbf{P}^f \mathbf{H}^T + \mathbf{R}\right)^{-1}. & (3) \end{aligned}$$

In the EnKF the forecast error covariance matrix is obtained through the ensemble of model forecast, using the relation

$$\mathbf{P}^f = \frac{1}{N-1} \sum_{i=1}^{N} \left(\mathbf{x}_i^f - \overline{\mathbf{x}}^f\right)\left(\mathbf{x}_i^f - \overline{\mathbf{x}}^f\right)^T, \tag{4}$$

where $\overline{\mathbf{x}}^f$ is the forecast ensemble average

- Ensemble least squares update is given by

$$\mathbf{x}_t^a = \mathbf{x}_t^f + \frac{\sigma_f^2}{\sigma_f^2 + \sigma_{\text{obs}}^2}(y_{\text{obs}} - \mathbf{x}_t^f)$$

- ENLS assumes diagonal $\mathbf{P}^f$
- Update applied only at pixels where an observation is made
- Update applied at each pixel separately for each ensemble member
- No accounting for model covariance structure between pixels

- State $\mathbf{x}_t$ vector of 180×360 pixel values
- Initial distribution from perturbed SOLIS-VSM Synoptic maps
- Worden-Harvey provides forward map
- SOLIS-VSM error provides observations and corresponding uncertainties
- N **forecast** ensemble members $\mathbf{x}_t^f$
- Observations, $y_{\rm obs}$, adjust $\mathbf{x}_t^f$ to form **analysis** ensemble $\mathbf{x}_t^a$
- Observation operator $\mathbf{H}[\mathbf{x}_t]$ restriction to Earth side of Sun

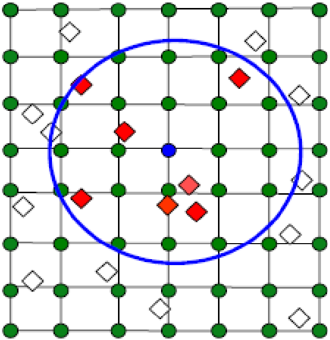


Figure : **Localization of observations.** Blue pixel being updated, red observations, green pixels in ellipse used for covariance structure.

- Small ensemble size $\implies$ spurious long distance correlations
- Update each pixel separately using only “local” observations
- $\mathbf{H}[\cdot]$ restriction to ellipse centered on pixel being updated
- Only y_{obs} inside ellipse used
- Localization region latitude dependent
- Determined by longitudinal spread

Data Assimilation

Ensemble Kalman Filter vs. Local Ensemble Kalman Filter

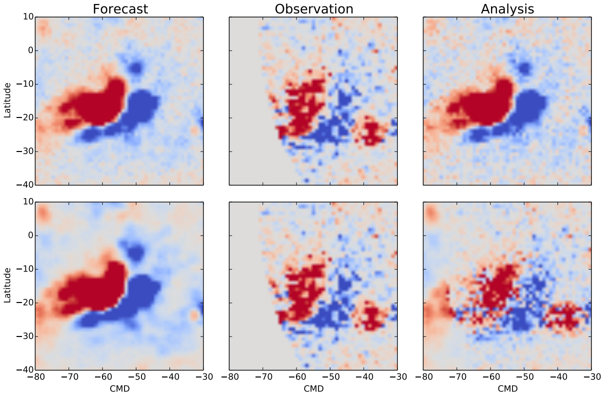


Figure : **Active region on East limb:** Forecast/Observation/Analysis ENKF (TOP) and LEKF (BOTTOM).

Data Assimilation
Ensemble Least Squares Filter vs. Local Ensemble Kalman Filter

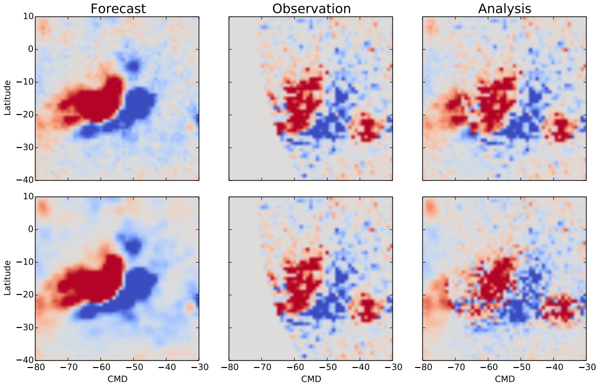


Figure : **Active region on East limb:** Forecast/Observation/Analysis ENLS (TOP) and LEKF (BOTTOM).

Data Assimilation

Effect of scale of observations

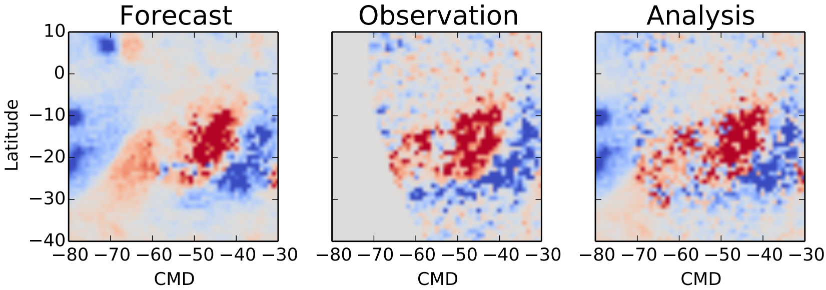


Figure : **Active region on East limb:** LETKF diffuses large scale structures.

Separation of Scales
Wavelet Decomposition

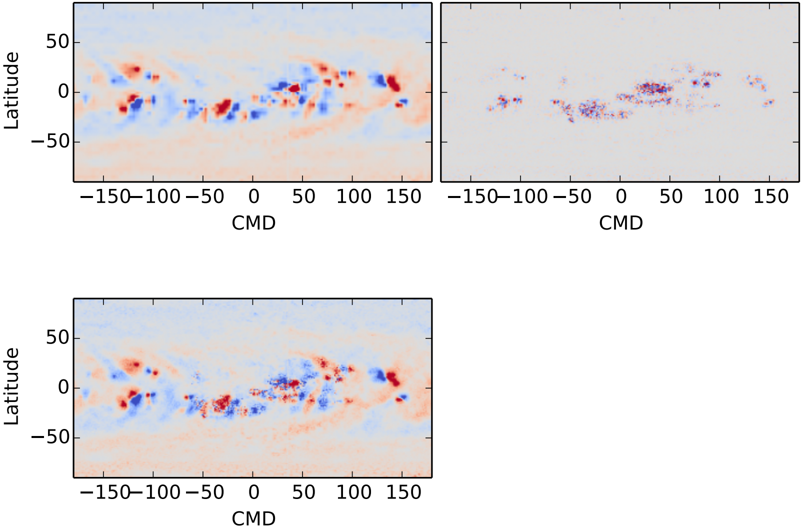
- **Cause:** Violation of *unbiased* assumption,

$$E[\mathbf{x}_t] = \mathbf{x}_{\mathrm{true}} \text{ and } E[y_{\mathrm{obs}}] = H[\mathbf{x}_{\mathrm{true}}]$$

- Linear decompositions work well with Kalman Filter
- Wavelet decomposition simple way to separate scales
- **Wavelet transform:** $\mathcal{W} y_{\mathrm{obs}}$
- **Decomposition projections:** Approximation $P_A \mathcal{W} y_{\mathrm{obs}}$ and detail $P_D \mathcal{W} y_{\mathrm{obs}}$
- **Reconstruction:**

$$y_{\mathrm{obs}} = \mathcal{W}^{-1} P_A \mathcal{W} y_{\mathrm{obs}} + \mathcal{W}^{-1} P_D \mathcal{W} y_{\mathrm{obs}}$$

Separation of Scales
Wavelet Decomposition



- With Gaussian assumption covariance transformed naturally
- $y_{\text{obs}} + \epsilon \sim N(y_{\text{obs}}, C_\epsilon)$
- $\mathcal{W}^{-1} P_A \mathcal{W}(y_{\text{obs}} + \epsilon)$ has covariance

$$\mathcal{W}^{-1} P_A \mathcal{W} C_\epsilon (\mathcal{W}^{-1} P_A \mathcal{W})^T$$

- Decomposition simultaneously on ensemble members $\mathbf{x}^f$
- Assimilation performed separately on each scale

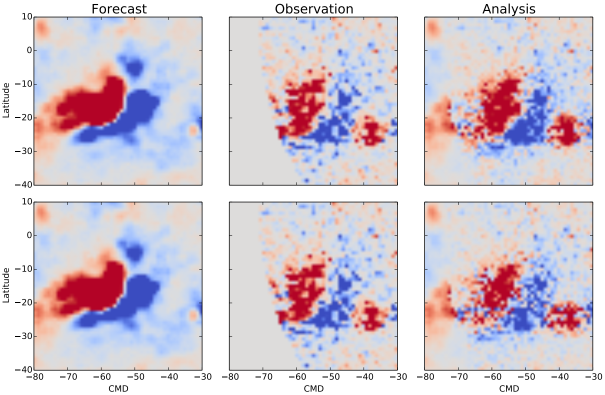


Figure : **MRletkf vs letkf:** Multi-resolution assimilation (TOP), LETKF (BOTTOM).

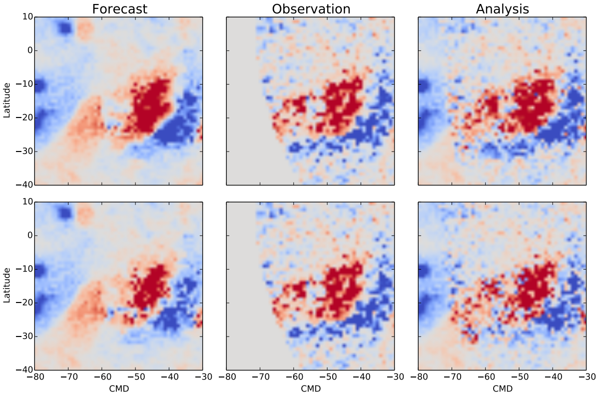


Figure : **MRletkf vs letkf:** Multi-resolution assimilation (TOP), LETKF (BOTTOM).

ADAPT assimilation

Conclusions/Future directions

- EnKF method applied to ADAPT to assimilate various observations
- assimilation does a good job, need to calibrate to get satisfactory results
- Preserve *physical* structure after assimilation, especially for active regions **(partially complete, developing and testing multi-resolution EnKF method)**
- Incorporate *smoothing* in assimilation, multiple observation times assimilated simultaneously
- Assimilate observations close to the poles of Sun; Combine assimilation of VSM and GONG observations to target poles