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Variance of High-frequency Precipitation

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Variance of High-frequency Precipitation

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Let $P_{in} \equiv$ precipitation rate [mm / hr] at i^{th} hour of n^{th} day. Then the daily mean rate is $\overline{P}_n^d \equiv 24^{-1} \sum_{i=1}^{24} P_{in}$ with $n = 1, 2, 3, \ldots, N$. The mean diurnal-cycle rate is $\overline{P}_i^c \equiv N^{-1} \sum_{n=1}^{N} P_{in}$ with $i = 1, 2, 3, \ldots, 24$. The overall mean including all hours and days is

$$\overline{P}^{ALL} \equiv (24\,N)^{-1} \sum_{i,n} P_{in} = N^{-1} \sum_n \overline{P}_n^d = 24^{-1} \sum_i \overline{P}_i^c \qquad [1]$$

Each of the three means has an associated variance, equal to an average of squares minus the square of an average. The variance over all hours and days is

$$\mathrm{Var}\left(P^{ALL}\right) \equiv (24\,N)^{-1} \sum_{i,n} (P_{in})^2 - (\overline{P}^{ALL})^2 \qquad [2]$$

The variance of daily means is

$$\mathrm{Var}\left(P^d\right) \equiv N^{-1} \sum_n (\overline{P}_n^d)^2 - \left(N^{-1} \sum_n \overline{P}_n^d\right)^2 \qquad [3]$$

$$= \left(24^2\,N\right)^{-1} \sum_{i,j,n} P_{in} P_{jn} - (\overline{P}^{ALL})^2$$

$$\Longrightarrow \quad \mathrm{Var}\left(P^{ALL}\right) - \mathrm{Var}\left(P^d\right) = (24\,N)^{-1} \sum_n \left(\sum_i (P_{in})^2 - 24^{-1} \sum_{i,j} P_{in} P_{jn}\right) \qquad [4]$$

If P were constant during each day then [4] would show that $\mathrm{Var}\left(P^{ALL}\right) = \mathrm{Var}\left(P^d\right)$. In general, however, $\mathrm{Var}\left(P^{ALL}\right) \geq \mathrm{Var}\left(P^d\right)$. The amount by which $\mathrm{Var}\left(P^{ALL}\right)$ exceeds $\mathrm{Var}\left(P^d\right)$ depends on the covariance between different hours of the day. If the timepoints were completely uncorrelated, then $\mathrm{Var}\left(P^d\right) = \mathrm{Var}\left(P^{ALL}\right)/24$ because variances add linearly in a random walk, but correlations will make $\mathrm{Var}\left(P^d\right)$ a larger fraction of $\mathrm{Var}\left(P^{ALL}\right)$.

Finally, the variance of diurnal-cycle means over the 24 hour cycle is

$$\mathrm{Var}(P^c) \equiv 24^{-1} \sum_i (\overline{P}_i^c)^2 - \left(24^{-1} \sum_i \overline{P}_i^c\right)^2 \qquad [5]$$

$$= \left(24\,N^2\right)^{-1} \sum_{i,n,m} P_{in} P_{im} - (\overline{P}^{ALL})^2$$

Note the difference between $\mathrm{Var}\left(P^d\right)$ and $\mathrm{Var}(P^c)$ shown by comparing [3] and [5].

$\mathrm{Var}\left(P^{ALL}\right)$, $\mathrm{Var}\left(P^d\right)$ and $\mathrm{Var}(P^c)$ are single numbers, in contrast to the variance (over days) at each hour of the diurnal cycle:

$$\mathrm{Var}\left(P^h\right)_i \equiv N^{-1} \sum_n (P_{in})^2 - (\overline{P}_i^c)^2 \text{ with } i = 1, 2, 3, \ldots, 24 \qquad [6]$$

Standard errors of diurnal cycle precipitation $\pm\sqrt{\mathrm{Var}\left(P^h\right)_i / N}$ are plotted as error bars by Covey et al. (2016, Figures 4 and S12).

Averaging [6] over the diurnal cycle and adding the result to [5], one finds that the $24^{-1} \sum_i (\overline{P}_i^c)^2$ terms cancel and, using [2],

$$\mathrm{Var}(P^c) + 24^{-1} \sum_i \mathrm{Var}\left(P^h\right)_i = (24\,N)^{-1} \sum_{i,n} (P_{in})^2 - (\overline{P}^{ALL})^2 = \mathrm{Var}\left(P^{ALL}\right) \qquad [7]$$

Thus $\mathrm{Var}\left(P^{ALL}\right)$ *can be divided into variance due to the mean diurnal cycle and variance due to the error bars on the diurnal cycle. This division is evidently different from the division of* $\mathrm{Var}\left(P^{ALL}\right)$ *into variance of daily means and a residual, as in [2]-[4].*

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